QCD at LHC

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QCD

Premio Nobel 2004!

- an unbroken Yang-Mills gauge field theory featuring asymptotic freedom $\longrightarrow$ confinement
- in non-perturbative regime (low Q^2) many approaches: lattice, Regge theory, χ PT, large N_c, HQET
- in perturbative regime (high Q^2) QCD is a precision toolkit for exploring Higgs & BSM physics
- LEP was an electroweak machine
- Tevatron & LHC are QCD machines

Precision QCD

Precise determination of

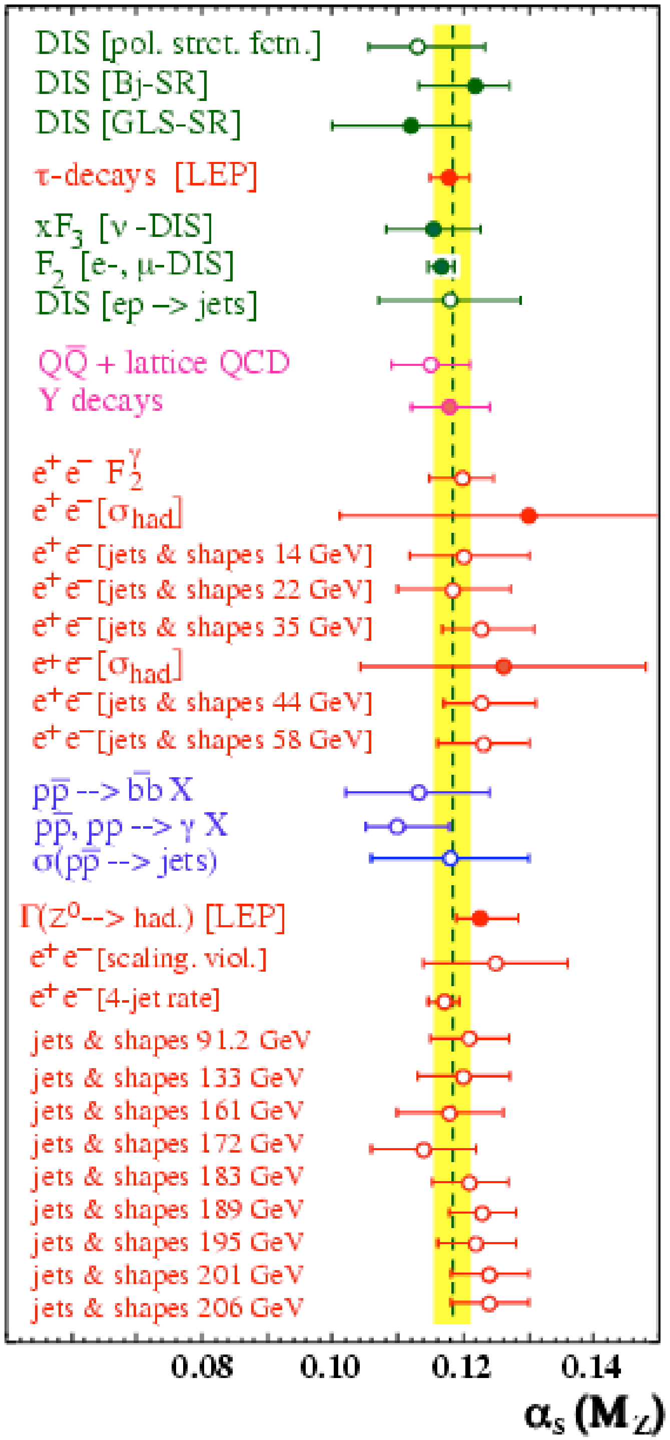
- strong coupling constant α_s
- parton distributions
- electroweak parameters
- LHC parton luminosity

Precise prediction for

- Higgs production
- new physics processes
- their backgrounds

Summary of $\alpha_S(M_Z)$

S. Bethke hep-ex/0407021



filled symbols are NNLO results

world average of $\alpha_S(M_Z)$
using $\overline{\mathrm{MS}}$ and NNLO results only

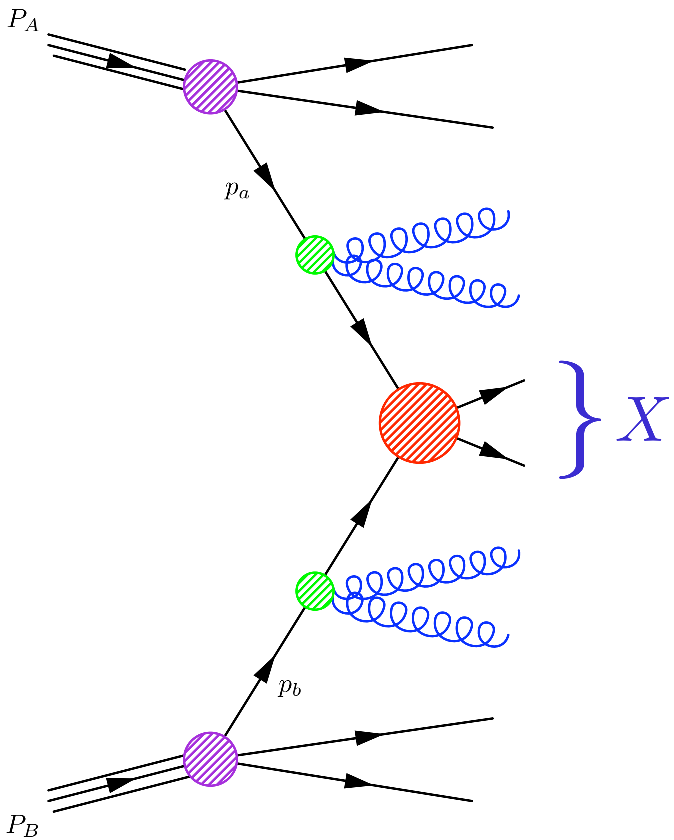
$$\alpha_S(M_Z) = 0.1182 \pm 0.0027$$

(cf. 2002 $\alpha_S(M_Z) = 0.1183 \pm 0.0027$
outcome almost identical
because new entries wrt 2002
- LEP jet shape observables and
4-jet rate, and HERA jet rates
and shape variables - are NLO)

Strong interactions at high Q^2

- Parton model
- Perturbative QCD
  - factorisation
  - universality of IR behaviour
  - cancellation of IR singularities
  - IR safe observables: inclusive rates
    - jets
    - event shapes

Factorisation



is the separation between
the short- and the long-range interactions

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{a/A}(x_1,\mu_F^2)\, f_{b/B}(x_2,\mu_F^2)$$
$$\times \quad \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

$$X = W, Z, H, Q\bar{Q}, \text{high-}E_T\text{jets}, \ldots$$

$\hat{\sigma}$ is known as a fixed-order expansion in α_S

$$\hat{\sigma} = C\alpha_S^n(1 + c_1\alpha_S + c_2\alpha_S^2 + \ldots)$$

$c_1 =$ NLO $\qquad c_2 =$ NNLO

or as an all-order resummation

$$\hat{\sigma} = C\alpha_S^n[1 + (c_{11}L + c_{10})\alpha_S + (c_{22}L^2 + c_{21}L + c_{20})\alpha_S^2 + \ldots]$$

where $L = \ln(M/q_T), \ln(1-x), \ln(1/x), \ln(1-T), \ldots$

$c_{11}, c_{22} =$ LL $\qquad c_{10}, c_{21} =$ NLL $\qquad c_{20} =$ NNLL

Evolution

- factorisation scale μ_F is arbitrary
  - cross section cannot depend on μ_F

$$\mu_F \frac{d\sigma}{d\mu_F} = 0$$

implies DGLAP equations

V. Gribov L. Lipatov; Y. Dokshitzer
G. Altarelli G. Parisi

$$\mu_F \frac{df_a(x,\mu_F^2)}{d\mu_F} = P_{ab}(x,\alpha_S(\mu_F^2)) \otimes f_b(x,\mu_F^2) + \mathcal{O}(\frac{1}{Q^2})$$

$$\mu_F \frac{d\hat{\sigma}_{ab}(Q^2/\mu_F^2,\alpha_S(\mu_F^2))}{d\mu_F} = -P_{ac}(x,\alpha_S(\mu_F^2)) \otimes \hat{\sigma}_{cb}(Q^2/\mu_F^2,\alpha_S(\mu_F^2)) + \mathcal{O}(\frac{1}{Q^2})$$

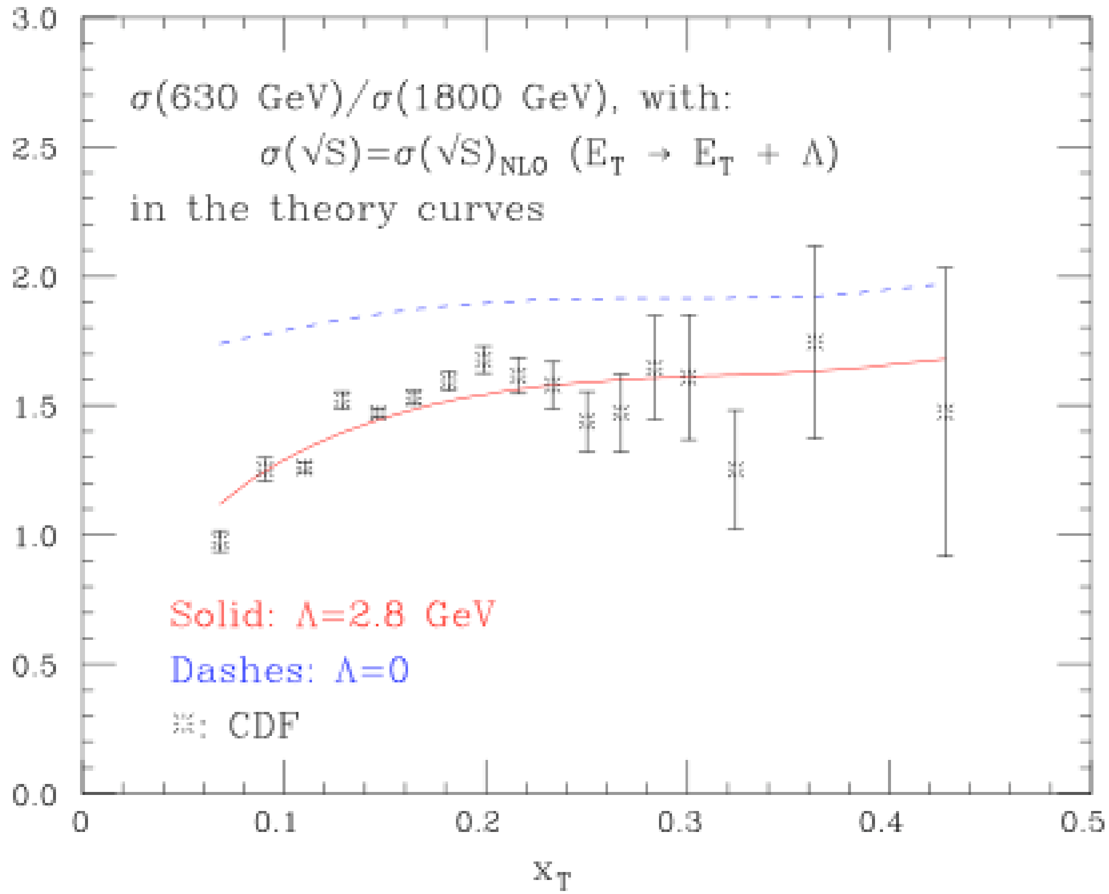
  - $P_{ab}(x,\alpha_S(\mu_F^2))$ is calculable in pQCD

Factorisation-breaking contributions

- underlying event (see Rick Field's studies at CDF)
- power corrections
  - MC's and theory modelling of power corrections laid out and tested at LEP where they provide an accurate determination of α_S models still need be tested in hadron collisions (see e.g. Tevatron studies at different $\sqrt{s}$) ⇨
- double-parton scattering
  - observed by Tevatron CDF in the inclusive sample $p\bar{p} \to \gamma + 3$ jets potentially important at LHC $\sigma_D \propto \sigma_S^2$
- diffractive events ⇨

Power corrections at Tevatron

Ratio of inclusive jet cross sections at 630 and 1800 GeV



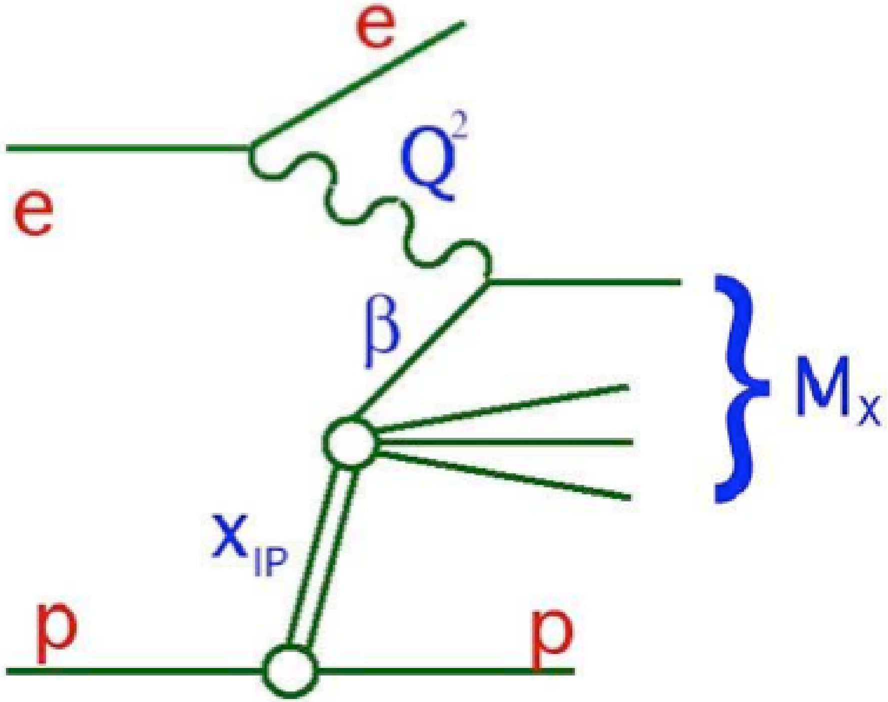
M.L. Mangano
KITP collider conf 2004

Bjorken-scaling variable

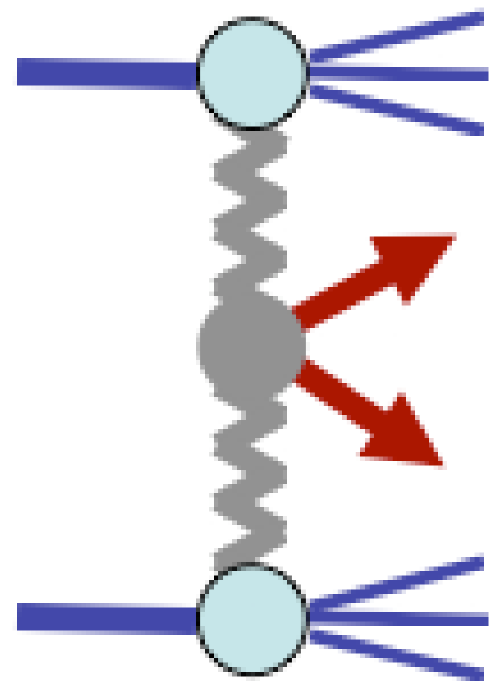
$$x_T = \frac{2E_T}{\sqrt{s}}$$

- In the ratio the dependence on the pdf's cancels
- dashes: theory prediction with no power corrections
- solid: best fit to data with free power-correction parameter Λ in the theory

Factorisation in diffraction ??



diffraction in DIS



double pomeron exchange in $p\bar{p}$

- no proof of factorisation in diffractive events
- data do not support it

3 complementary approaches to $\hat{\sigma}$

	matrix-elem MC's	fixed-order x-sect	shower MC's
final-state description	hard-parton jets. Describes geometry, correlations, ...	limited access to final-state structure	full information available at the hadron level
higher-order effects: loop corrections	hard to implement: must introduce negative probabilities	straightforward to implement (when available)	included as vertex corrections (Sudakov FF's)
higher-order effects: hard emissions	included, up to high orders (multijets)	straightforward to implement (when available)	approximate, incomplete phase space at large angles
resummation of large logs	?	feasible (when available)	unitarity implementation (i.e. correct shapes but not total rates)

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Matrix-element MonteCarlo generators

- efficient multi-parton generation: up to 2 ➡ 9 jets subprocesses
  - ALPGEN M.L.Mangano M. Moretti F. Piccinini R. Pittau A. Polosa 2002
  - MADGRAPH/MADEVENT W.F. Long F. Maltoni T. Stelzer 1994/2003
  - COMPHEP A. Pukhov et al. 1999
  - GRACE/GR@PPA T. Ishikawa et al. K. Sato et al. 1992/2001
  - HELAC C. Papadopoulos et al. 2000

- merged with parton showers
  - all of the above, merged with HERWIG or PYTHIA
  - SHERPA F. Krauss et al. 2003

➡ talk di Frixione

Shower MonteCarlo generators

- HERWIG B. Webber et al. 1992

  being re-written as a C++ code (HERWIG++)

- PYTHIA T. Sjostrand 1994

and more

- CKKW S. Catani F. Krauss R. Kuhn B. Webber 2001

  a procedure to interface parton subprocesses with
  a different number of final states to parton showers

- MC@NLO S. Frixione B. Webber 2002

  a procedure to interface NLO computations to shower MC's

→ talk di Frixione

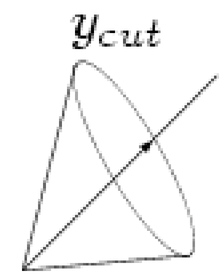
NLO features

- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: μ_R, μ_F
    - predictive normalisation of observables
        - first step toward precision measurements
        - accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC's: MC@NLO

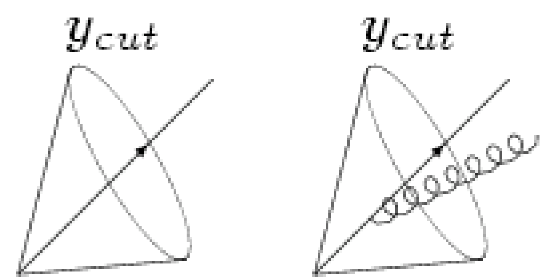
Jet structure

the jet non-trivial structure shows up first at NLO

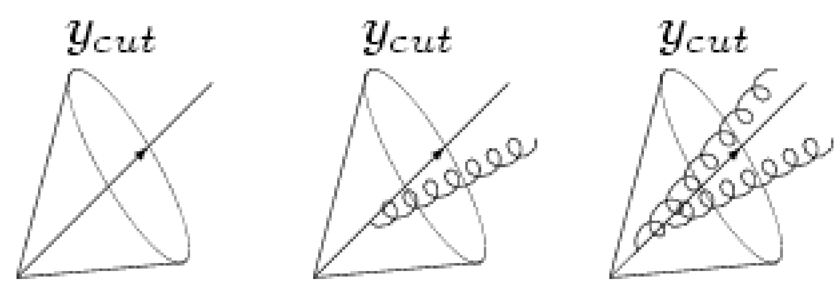
leading order



NLO



NNLO



Somebody's wishlist

Dear Santa Claus,

I'd like to have the following cross sections at NLO

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}+\leq 3j$	$WW+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}+\leq 3j$	$WW+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$ZZ+b\bar{b}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{b}+\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$		$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}+\leq 3j$	$\gamma\gamma+b\bar{b}+\leq 3j$		
$\gamma+c\bar{c}+\leq 3j$	$\gamma\gamma+c\bar{c}+\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}+\leq 3j$		
	$WZ+c\bar{c}+\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

NLO history

- $e^+e^- \to 3$ jets — K. Ellis, D. Ross, A. Terrano 1981
  $e^+e^- \to 4$ jets — Z. Bern et al., N. Glover et al., Z. Nagy Z.Trocsanyi 1996-97
- $pp \to 1, 2$ jets — K. Ellis J. Sexton 1986, W. Giele N. Glover D. Kosower 1993
  $pp \to 3$ jets — Z. Bern et al., Z. Kunszt et al. 1993-1995, Z. Nagy 2001
- $pp \to V + 1$ jet — W. Giele N. Glover & D. Kosower 1993
  $pp \to V + 2$ jet — Bern et al., Glover et al. 1996-97, K. Ellis & Campbell 2003
- $pp \to V b\bar{b}$ — K. Ellis & J. Campbell 2003
  $pp \to V b\bar{b} + 1$ jet — ??
- $pp \to VV$ — Ohnemus & Owens, Baur et al. 1991-96, Dixon et al. 2000
  $pp \to VV + 1$ jet — ??
- $pp \to \gamma\gamma$ — B. Bailey et al 1992, T. Binoth et al 1999
  $pp \to \gamma\gamma + 1$ jet — Z. Bern et al. 1994, V. Del Duca et al. 2003
- $pp \to Q\bar{Q}$ — Dawson K. Ellis Nason 1989, Mangano Nason Ridolfi 1992
  $pp \to Q\bar{Q} + 1$ jet — A. Brandenburg et al. 2005 ?

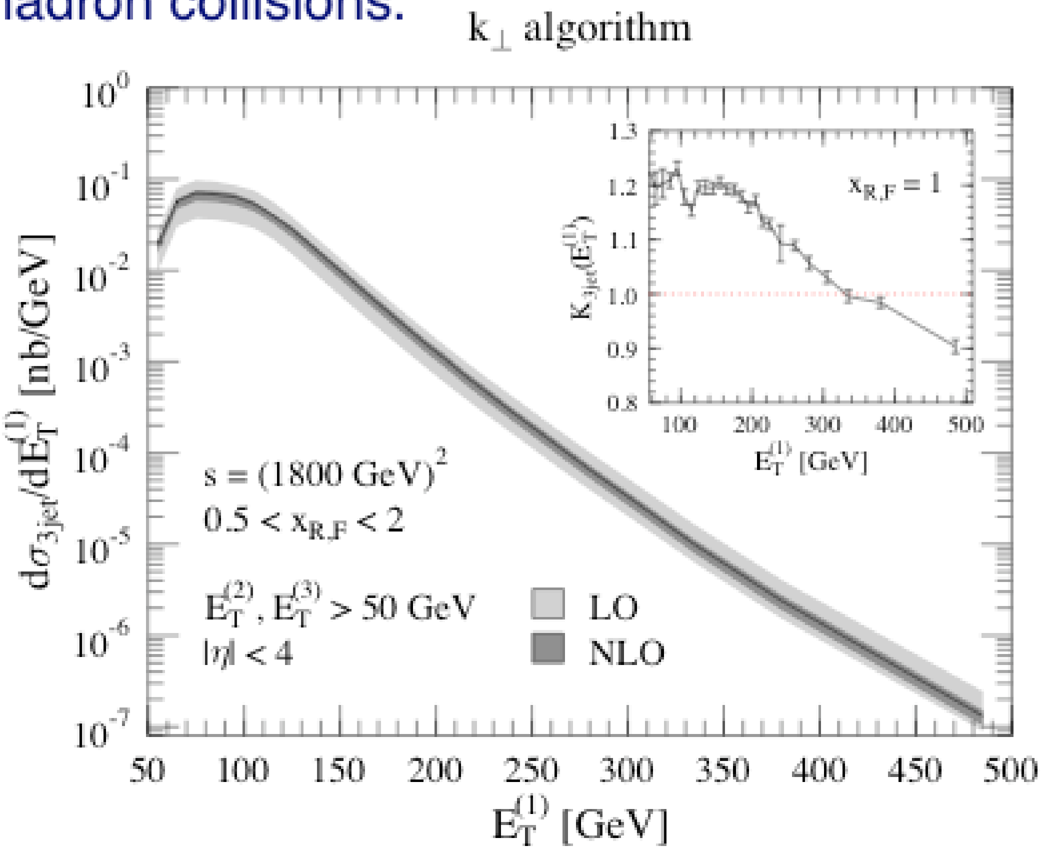
NLOJET++

Author(s): Z. Nagy
http://www.ippp.dur.ac.uk/~nagyz/nlo++.html
Multi-purpose C++ library for calculating jet cross-sections in e^+e^- annihilation, DIS and hadron-hadron collisions.

$e^+e^- \longrightarrow \quad \leq 4$ jets

$ep \longrightarrow (\leq 3+1)$ jets

$p\bar{p} \longrightarrow \quad \leq 3$ jets



hep-ph/0110315

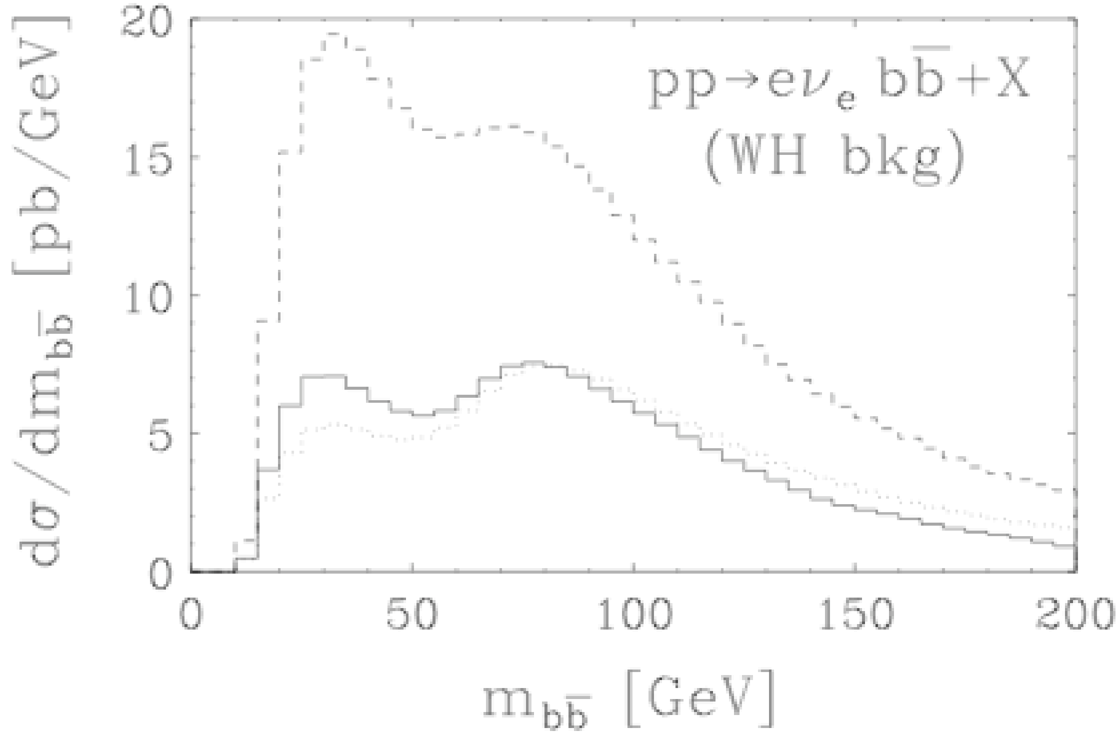
MCFM

Author(s): JC, R. K. Ellis
http://mcfm.fnal.gov
Fortran package for calculating a number of processes involving vector bosons, Higgs, jets and heavy quarks at hadron colliders.

$p\bar{p} \longrightarrow V + \leq 2$ jets

$p\bar{p} \longrightarrow V + b\bar{b}$

with $V = W, Z$.



hep-ph/0308195

AYLEN/EMILIA

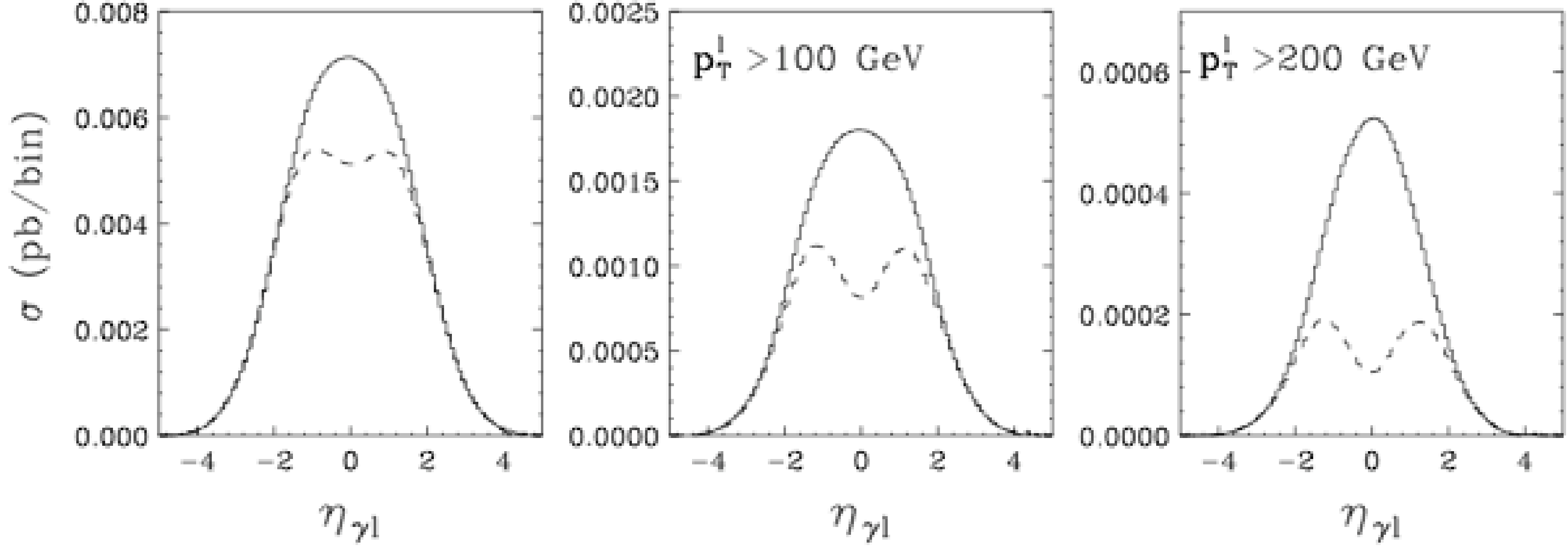
Author(s): L. Dixon, Z. Kunszt, A.Signer, D. de Florian

http://www.itp.phys.ethz.ch/staff/dflorian/codes.html

Fortran implementation of gauge boson pair production at hadron colliders, including full spin and decay angle correlations.

$p\bar{p} \longrightarrow VV'$ and $p\bar{p} \longrightarrow V\gamma$ with $V, V' = W, Z$

Anomalous triple gauge boson couplings at the LHC:



hep-ph/0002138

DIPHOX/EPHOX

Author(s): P. Aurenche, T.Binoth, M. Fontannaz, J. Ph. Guillet, G. Heinrich, E. Pilon, M. Werlen

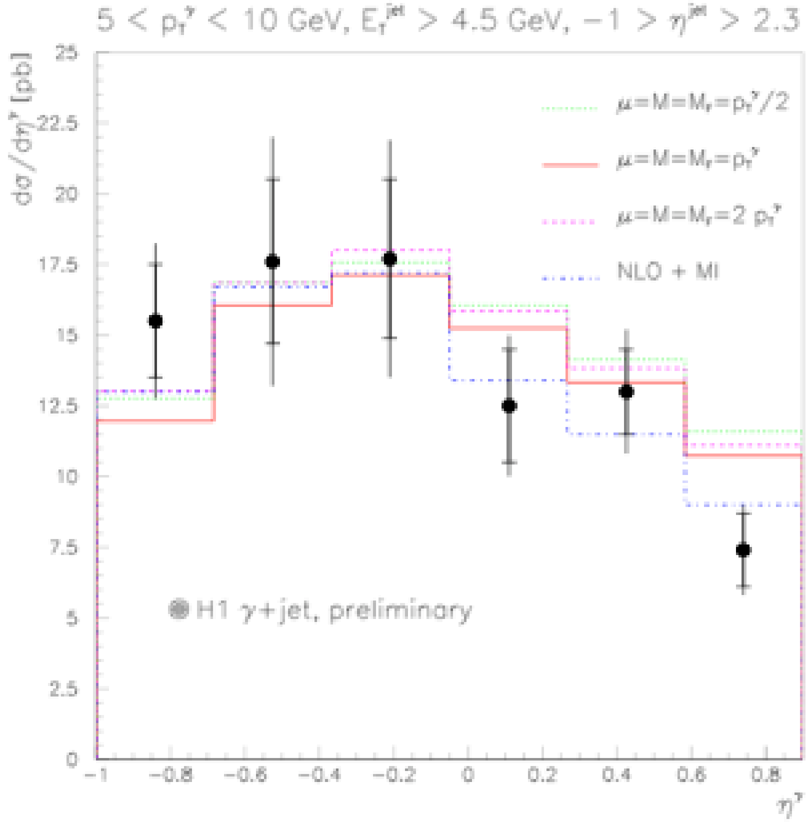
http://wwwlapp.in2p3.fr/lapth/PHOX_FAMILY/main.html

Fortran code to compute processes involving photons, hadrons and jets in DIS and hadron colliders.

$$p\bar{p} \longrightarrow \gamma + \leq 1 \text{ jet}$$

$$p\bar{p} \longrightarrow \gamma\gamma$$

$$\gamma p \longrightarrow \gamma + \text{jet}$$

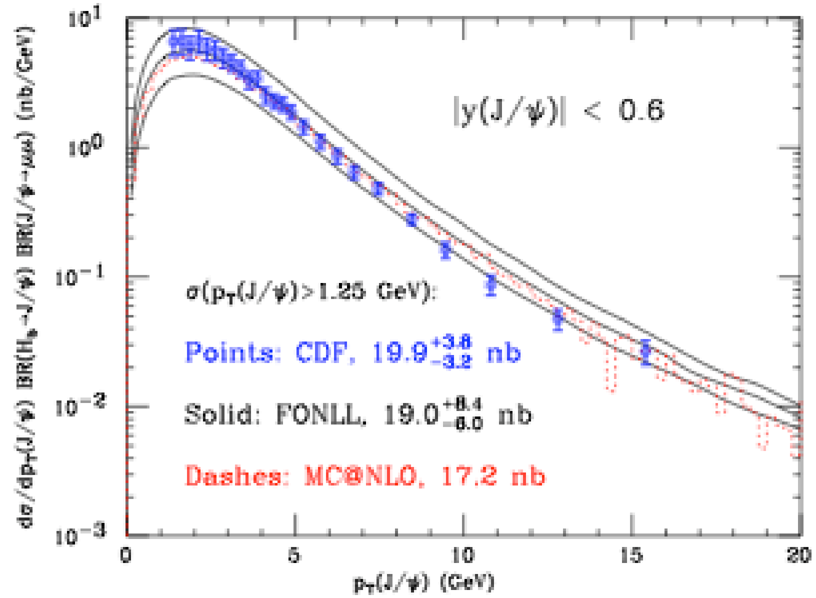


Preliminary H1 data, hep-ph/0312070.

Heavy quark production

Author(s): M. L. Mangano, P. Nason and G. Ridolfi
http://www.ge.infn.it/~ridolfi/hvqlibx.tgz
Fortran code for the calculation of heavy quark cross-sections and distributions in a fully differential manner

- Based on the more inclusive calculations of Dawson et al, Beenakker et al.
- Does not include multiple gluon radiation, $\log(p_T/m_b)$ (FONLL)
  Cacciari et al., hep-ph/9803400
- These are the same matrix elements that are incorporated into MC@NLO
  Frixione et al., hep-ph/0305252

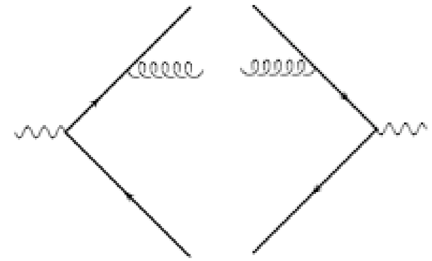


hep-ph/0312132

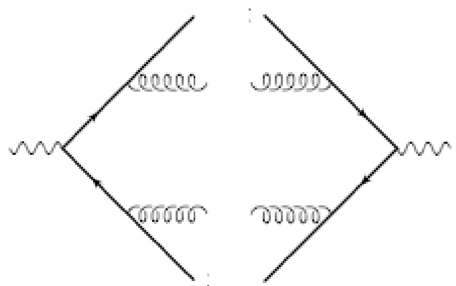
NLO assembly kit

$e^+e^- \to 3$ jets

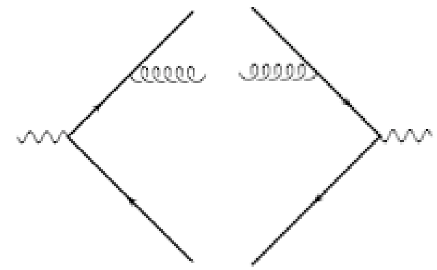
leading order $\quad |\mathcal{M}_n^{tree}|^2$



NLO real



IR

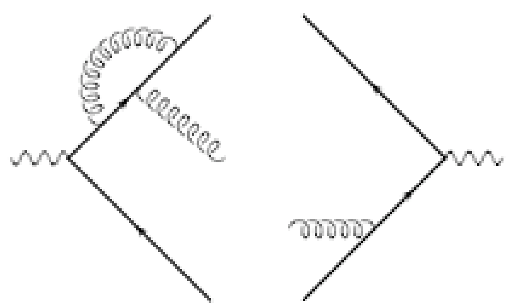


$\otimes$

$$|\mathcal{M}_{n+1}^{tree}|^2 \quad \to \quad |\mathcal{M}_n^{tree}|^2 \quad + \quad \int dPS |P_{split}|^2$$

$$= \quad -\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)$$

NLO virtual



$d = 4 - 2\epsilon$

$$\int d^d l \; 2(\mathcal{M}_n^{loop})^* \mathcal{M}_n^{tree} \quad = \quad \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right) |\mathcal{M}_n^{tree}|^2 + fin.$$

NLO production rates

Process-independent procedure devised in 1992-96

Giele Glover & Kosower; Frixione Kunszt & Signer, Catani & Seymour

slicing subtraction

$$\hat{\sigma} = \sigma^{\mathrm{LO}} + \sigma^{\mathrm{NLO}} = \int_n d\sigma^B + \sigma^{\mathrm{NLO}}$$

$$\sigma^{\mathrm{NLO}} = \int_{n+1} d\sigma^R + \int_n d\sigma^V$$

the 2 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

$$\sigma^{\mathrm{NLO}} = \int_{n+1} \left[(d\sigma^R)_{\epsilon=0} - (d\sigma^A)_{\epsilon=0} \right] + \int_n \left(d\sigma^V + \int_1 d\sigma^A \right)_{\epsilon=0}$$

the 2 terms on the rhs are finite in d=4

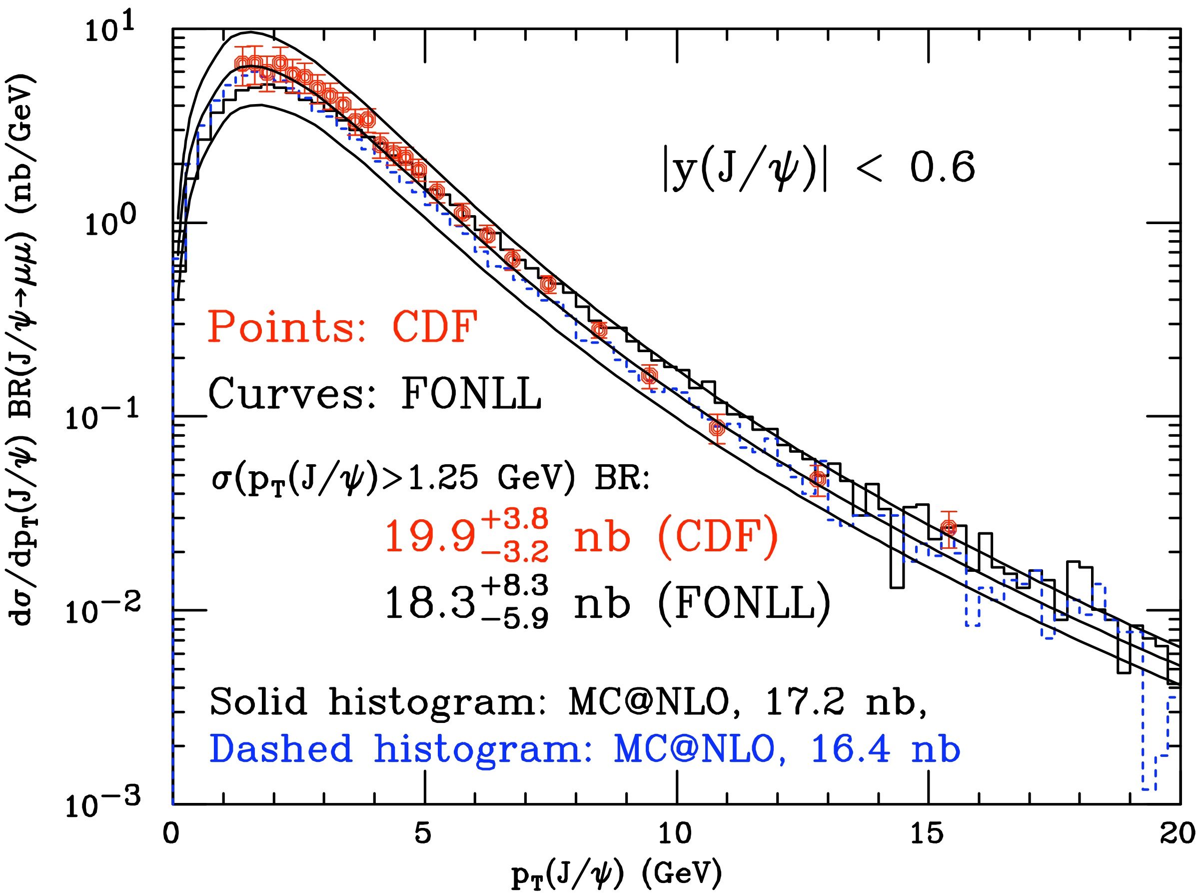
NLO complications

- loop integrals are involved and process-dependent
- more particles ➡ many scales ➡ lenghty analytic expressions
  - even though it is known how to compute loop integrals with $2 \to n$ particles no integrals with $n > 3\ (4)$ have been computed analytically (numerically)
- no numeric methods yet for hadron collisions
  - counterterms are subtracted analytically

Is NLO enough to describe data ?

b cross section in $p\bar{p}$ collisions at 1.96 TeV

$d\sigma(p\bar{p} \to H_b X, H_b \to J/\psi\ X)/dp_T(J/\psi)$



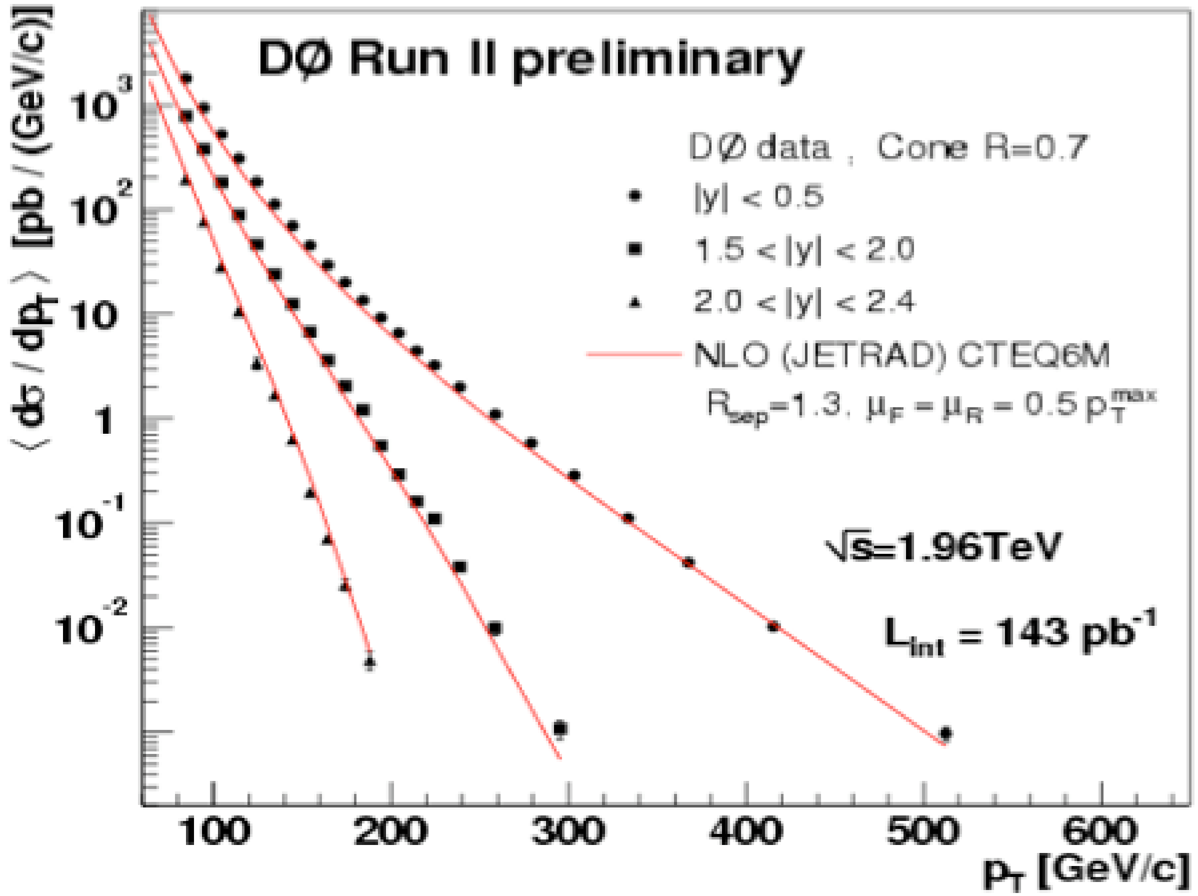
NLO + NLL

perfect agreement with data (with use of updated FF's by Cacciari & Nason)

Cacciari, Frixione, Mangano, Nason, Ridolfi 2003

Is NLO enough to describe data ?

Inclusive jet p_T cross section at Tevatron

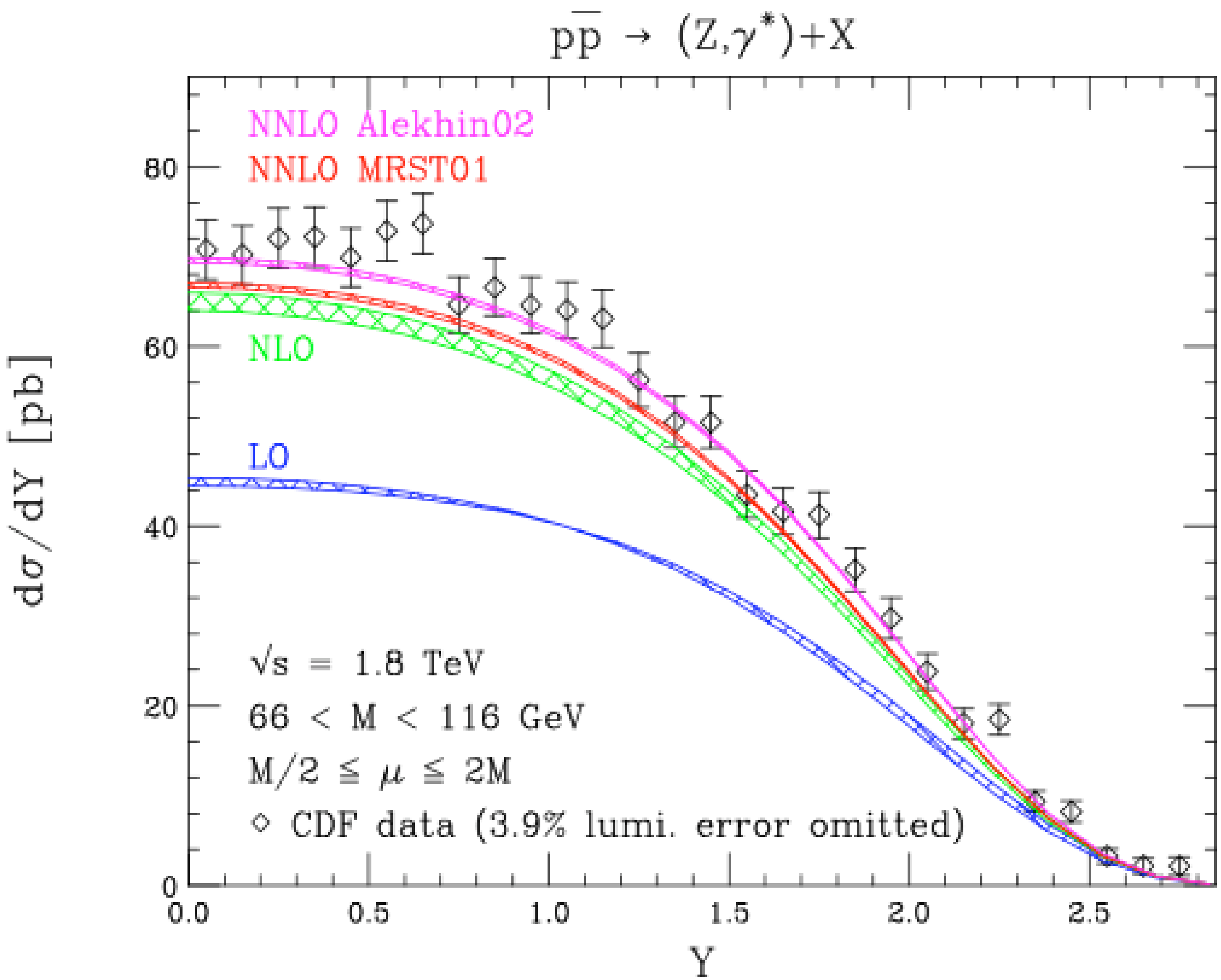


good agreement between NLO and data over several orders of magnitude

constrains the gluon distribution at high x

Is NLO enough to describe data ?

di-lepton rapidity distribution for (Z, γ^*) production vs. Tevatron Run I data



LO and NLO curves are for the MRST PDF set

no spin correlations

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Is NLO enough to describe data ?

Drell-Yan W cross section at LHC with leptonic decay of the W

Cuts A $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 20$ GeV, $p_T^{(\nu)} > 20$ GeV

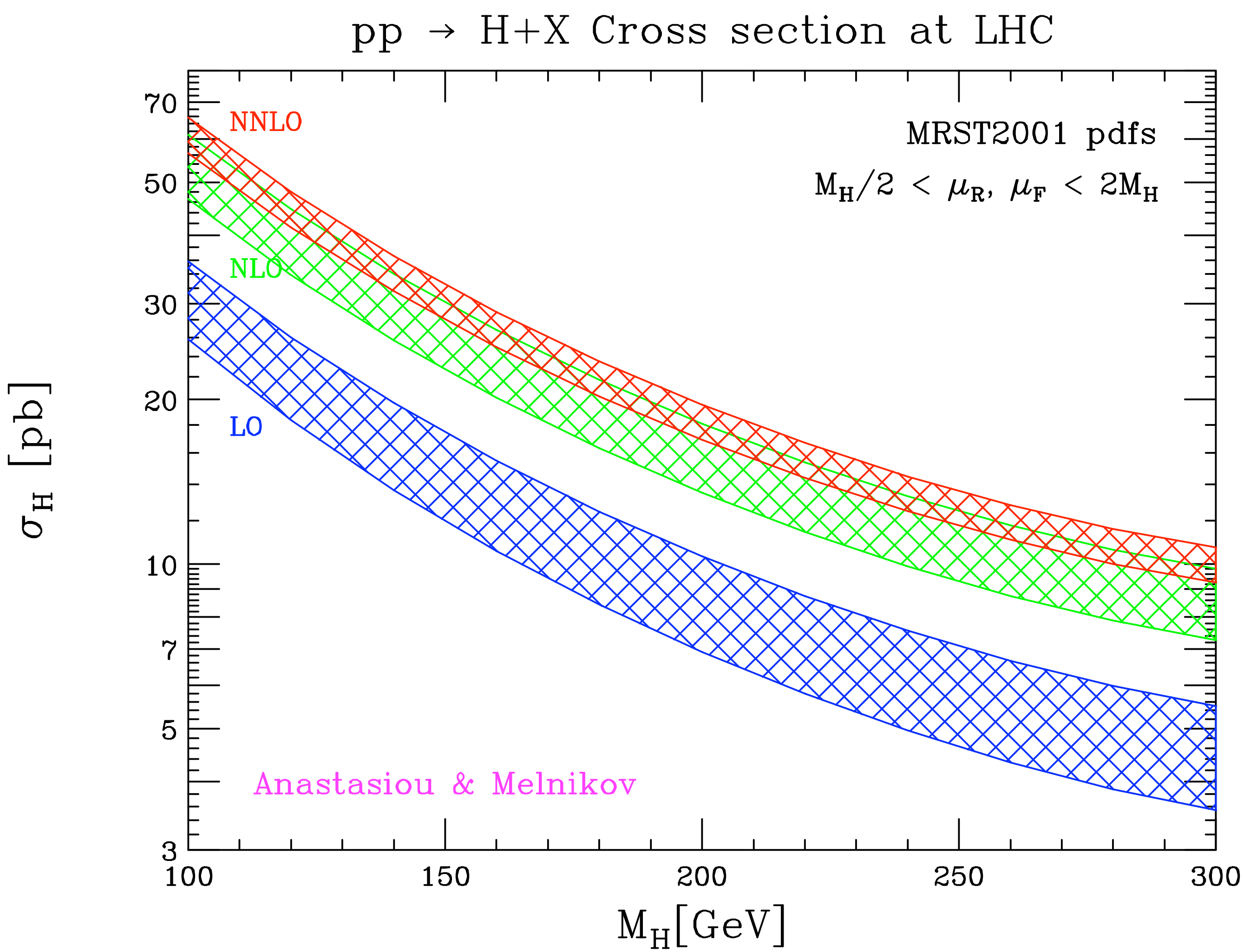
Cuts B $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 40$ GeV, $p_T^{(\nu)} > 20$ GeV

	LO		LO+HW	NLO		MC@NLO
Cuts A	0.5249	$\xrightarrow{-7.7\%}$	0.4843	0.4771	$\xrightarrow{+1.5\%}$	0.4845
	↓5.4%			↓7.0%		↓6.3%
Cuts A, no spin	0.5535			0.5104		0.5151
Cuts B	0.0585	$\xrightarrow{+208\%}$	0.1218	0.1292	$\xrightarrow{+2.9\%}$	0.1329
	↓29%			↓16%		↓18%
Cuts B, no spin	0.0752			0.1504		0.1570

- $|\mathrm{MC@NLO} - \mathrm{NLO}| = \mathcal{O}(2\%)$ — S. Frixione M.L. Mangano 2004
- NNLO useless without spin correlations
- Precisely evaluated Drell-Yan W, Z cross sections could be used as ``standard candles'' to measure the parton luminosity at LHC

Is NLO enough to describe data ?

Total cross section for inclusive Higgs production at LHC



contour bands are

lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$

upper

$\mu_R = M_H/2 \quad \mu_F = 2M_H$

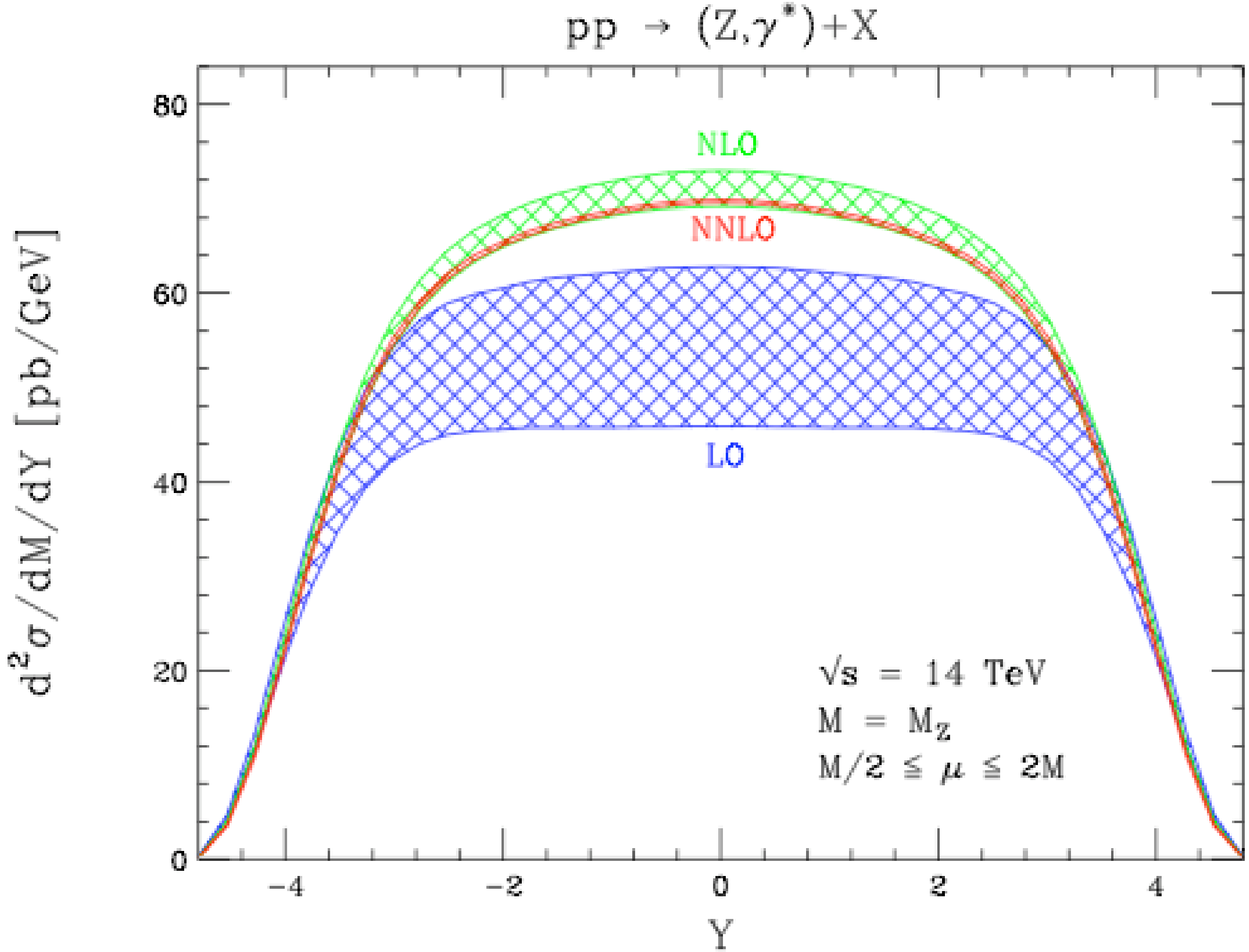
scale uncertainty is about 10%

NNLO prediction stabilises the perturbative series

NNLO state of the art

- Drell-Yan W, Z production
  - total cross section — Hamberg, van Neerven, Matsuura 1990; Harlander, Kilgore 2002
  - rapidity distribution — Anastasiou et al. 2003
- Higgs production
  - total cross section — Harlander, Kilgore; Anastasiou, Melnikov 2002
  - fully differential cross section — Anastasiou, Melnikov, Petriello 2004
- $e^+e^- \to 3$ jets
  - the C_F^2 term — the Gehrmanns, Glover 2004

Drell-Yan Z production at LHC



Rapidity distribution for an on-shell Z boson

- $30\%(15\%)$ NLO increase wrt to LO at central Y's (at large Y's) NNLO decreases NLO by $1-2\%$
- scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at NLO; less than 1% at NNLO

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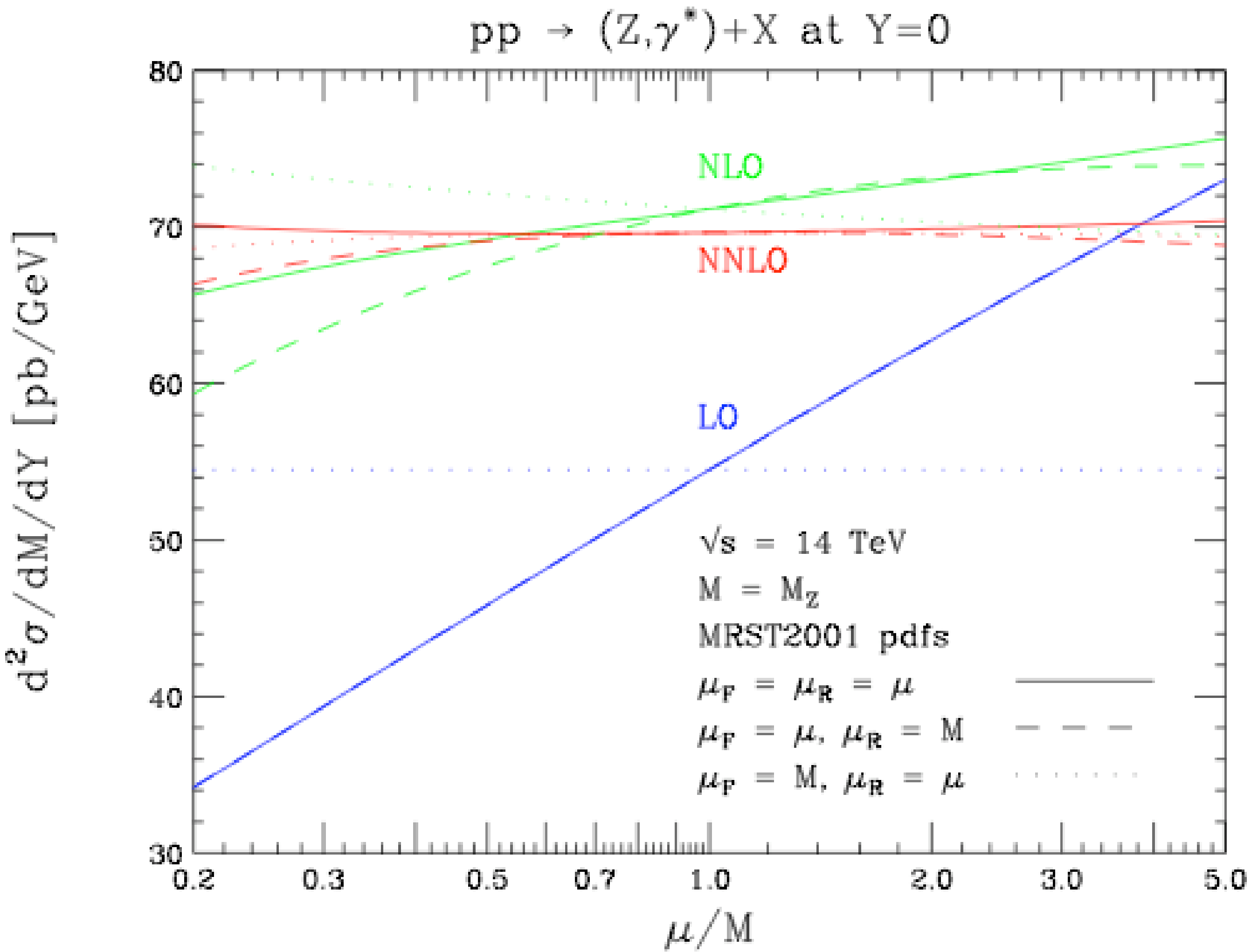
Scale variations in Drell-Yan Z production



- solid: vary μ_R and μ_F together
- dashed: vary μ_F only
- dotted: vary μ_R only

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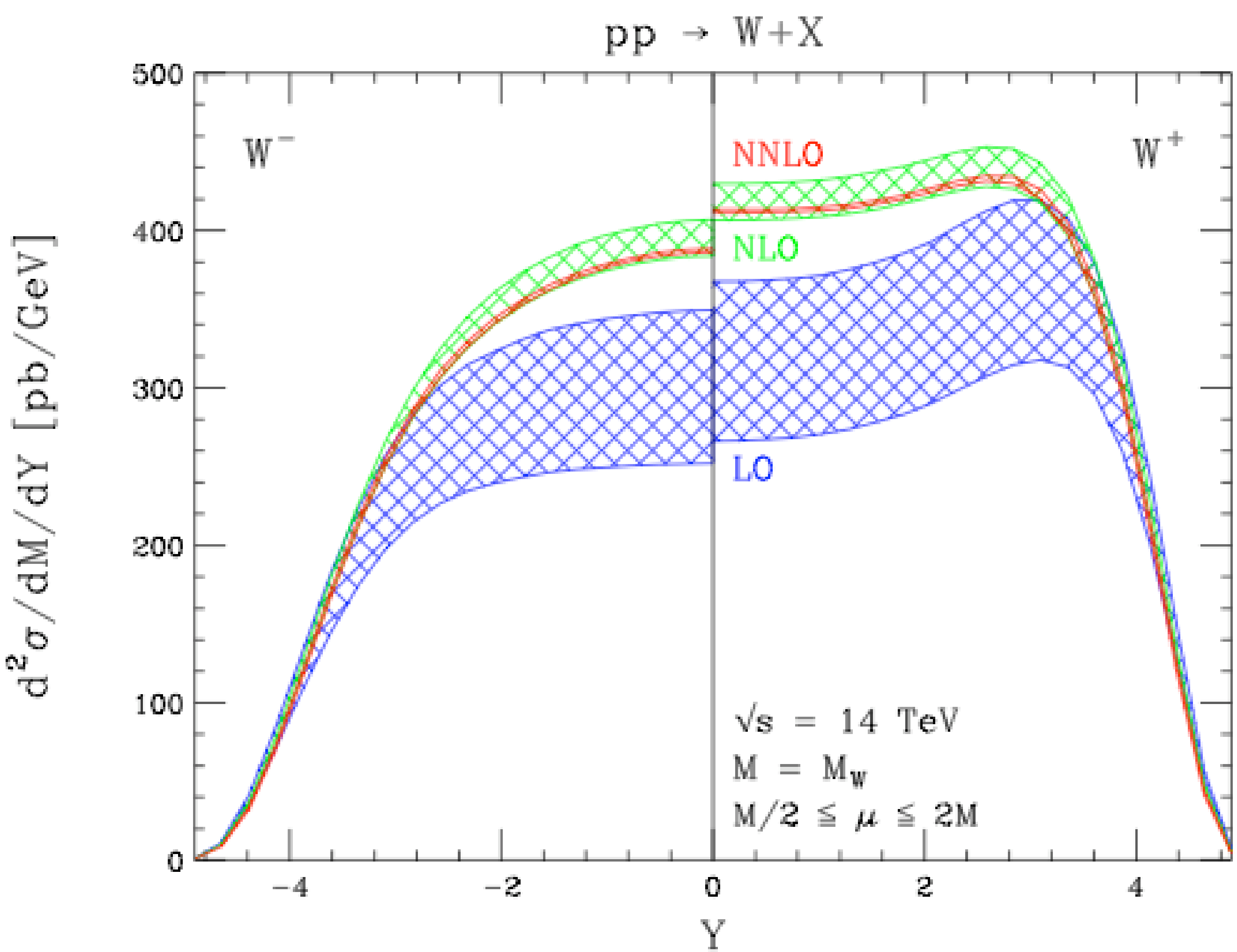
Drell-Yan W production at LHC



Rapidity distribution for an on-shell W^- boson (left) W^+ boson (right)

- distributions are symmetric in Y
- NNLO scale variations are $1\%(3\%)$ at central (large) Y

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Higgs production at LHC

a fully differential cross section:
bin-integrated rapidity distribution, with a jet veto

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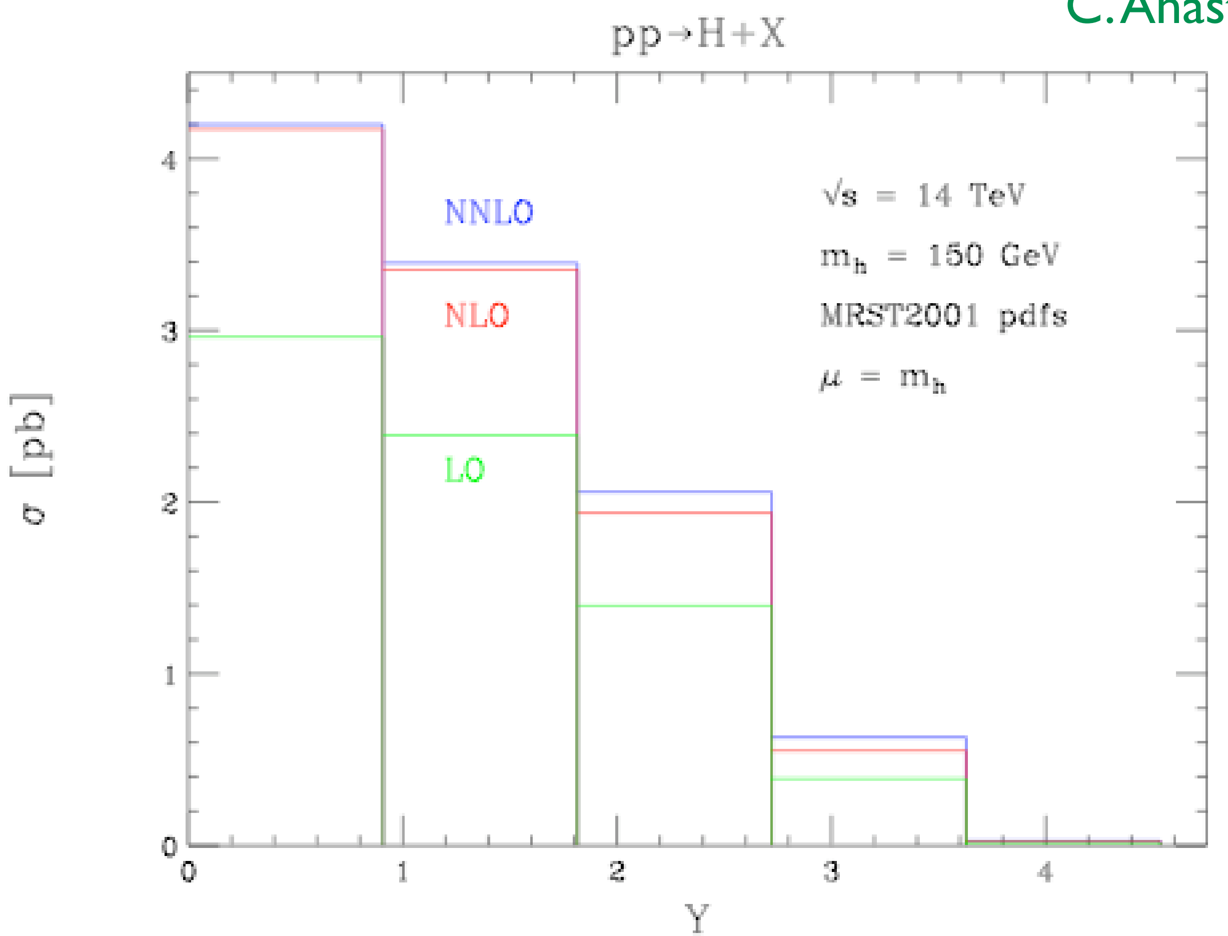


jet veto: require

$R = 0.4$

$|\mathbf{p}_T^j| < p_T^{veto} = 40 \text{ GeV}$

for 2 partons

$R_{12}^2 = (\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2$

if $R_{12} > R$

$|\mathbf{p}_T^1|, |\mathbf{p}_T^2| < p_T^{veto}$

if $R_{12} < R$

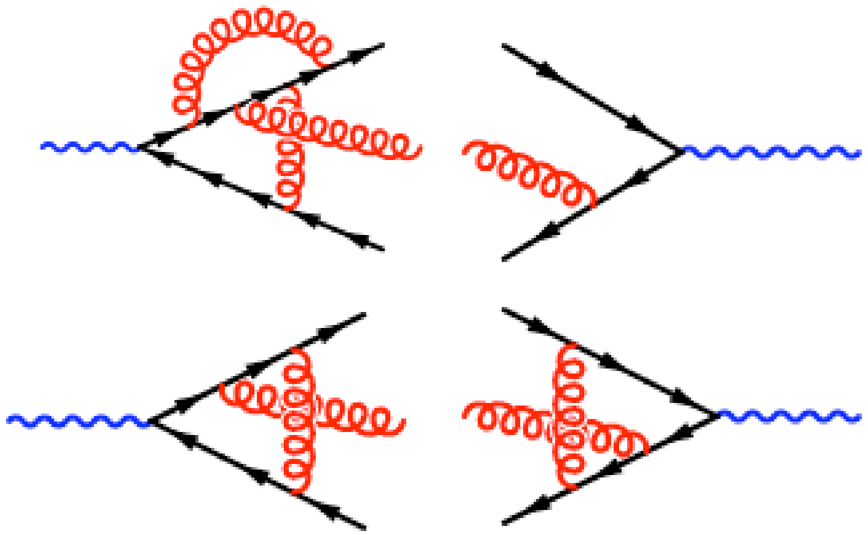
$|\mathbf{p}_T^1 + \mathbf{p}_T^2| < p_T^{veto}$

- $M_H = 150 \text{ GeV}$ (jet veto relevant in the $H \to W^+W^-$ decay channel)
- K factor is much smaller for the vetoed x-sect than for the inclusive one: average $|\mathbf{p}_T^j|$ increases from NLO to NNLO: less x-sect passes the veto

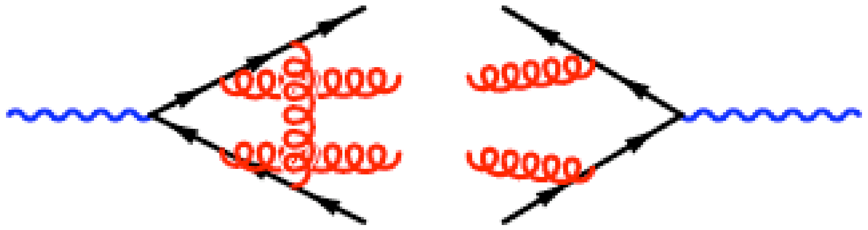
NNLO assembly kit

$e^+e^- \to 3$ jets

double virtual



real-virtual



double real

Two-loop matrix elements

- two-jet production $qq' \to qq', \; q\bar{q} \to q\bar{q}, \; q\bar{q} \to gg, \; gg \to gg$

  C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01

  Z. Bern A. De Freitas L. Dixon 2002

- photon-pair production $q\bar{q} \to \gamma\gamma, \; gg \to \gamma\gamma$

  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
  Z. Bern A. De Freitas L. Dixon 2002

- $e^+e^- \to 3$ jets $\qquad \gamma^* \to q\bar{q}g$

  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

- $V + 1$ jet production $q\bar{q} \to Vg$

  T. Gehrmann E. Remiddi 2002

- Drell-Yan V production $q\bar{q} \to V$

  R. Hamberg W. van Neerven T. Matsuura 1991

- Higgs production $gg \to H$ (in the $m_t \to \infty$ limit)

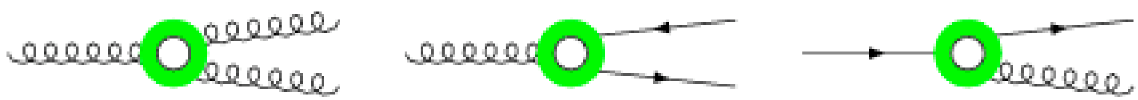
  R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

NNLO cross sections

- universal IR structure ⟹ process-independent procedure
- universal collinear and soft currents
  - 3-parton tree splitting functions

J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

  - 2-parton one-loop splitting functions



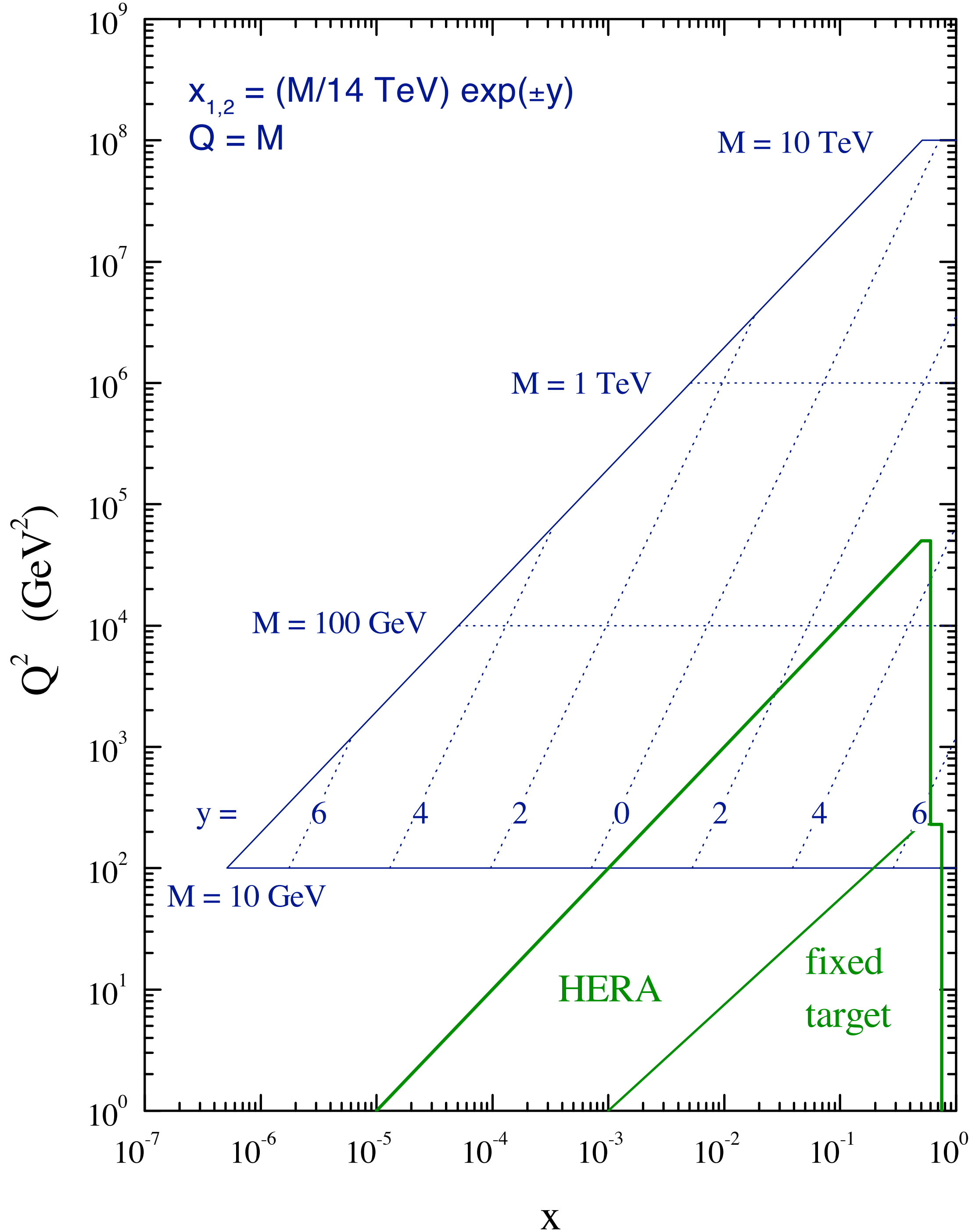
Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; D. Kosower 2003

- universal subtraction counterterms
  - several ideas and works in progress
    but so far not yet completely figured out

S. Weinzierl; A. Gehrmann-De Ridder T. Gehrmann G. Heinrich 2003

LHC parton kinematics

J. Stirling



Parton distribution functions (PDF)

- factorisation for the structure functions (e.g. $F_2^{ep}, \; F_L^{ep}$)

$$\mathcal{F}_i(x,\mu_F^2) = C_{ij} \otimes q_j + C_{ig} \otimes g$$

with the convolution $$[a \otimes b](x) \equiv \int_x^1 \frac{dy}{y}\, a(y)\, b\left(\frac{x}{y}\right)$$

$C_{ij}, \; C_{ig}$ coefficient functions

$q_i(x,\mu_F^2) \quad g(x,\mu_F^2)$ PDF's

- DGLAP evolution equations

$$\frac{d}{d\ln\mu_F^2}\begin{pmatrix} q_i \\ g \end{pmatrix} = \begin{pmatrix} P_{\mathrm{q_i q_j}} & P_{\mathrm{q_j g}} \\ P_{\mathrm{g q_j}} & P_{\mathrm{gg}} \end{pmatrix} \otimes \begin{pmatrix} q_j \\ g \end{pmatrix}$$

- perturbative series $P_{ij} \approx \alpha_s P_{ij}^{(0)} + \alpha_s^2 P_{ij}^{(1)} + \alpha_s^3 P_{ij}^{(2)}$

- anomalous dimension $\gamma_{ij}(N) = -\int_0^1 dx\; x^{N-1}\; P_{ij}(x)$

PDF's

- general structure of the quark-quark splitting functions

$$P_{q_i q_k} = P_{\bar{q}_i \bar{q}_k} = \delta_{ik} P^{v}_{qq} + P^{s}_{qq}$$
$$P_{q_i \bar{q}_k} = P_{\bar{q}_i q_k} = \delta_{ik} P^{v}_{q\bar{q}} + P^{s}_{q\bar{q}}$$

- non-singlet
  - flavour asymmetry

$$q^{\pm}_{ns,ik} = q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k) \quad \Longleftarrow \quad P^{\pm}_{ns} = P^{v}_{qq} \pm P^{v}_{q\bar{q}}$$

  - sum of valence distributions of all flavours

$$q^{v}_{ns} = \sum_{r=1}^{n_f} (q_r - \bar{q}_r) \quad \Longleftarrow \quad P^{v}_{ns} = P^{v}_{qq} - P^{v}_{q\bar{q}} + n_f (P^{s}_{qq} - P^{s}_{q\bar{q}})$$

- singlet

$$q_s = \sum_{i=1}^{n_f} (q_i + \bar{q}_i) \quad \Longleftarrow \quad \frac{d}{d \ln \mu_F^2} \begin{pmatrix} q_s \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q_s \\ g \end{pmatrix}$$

with

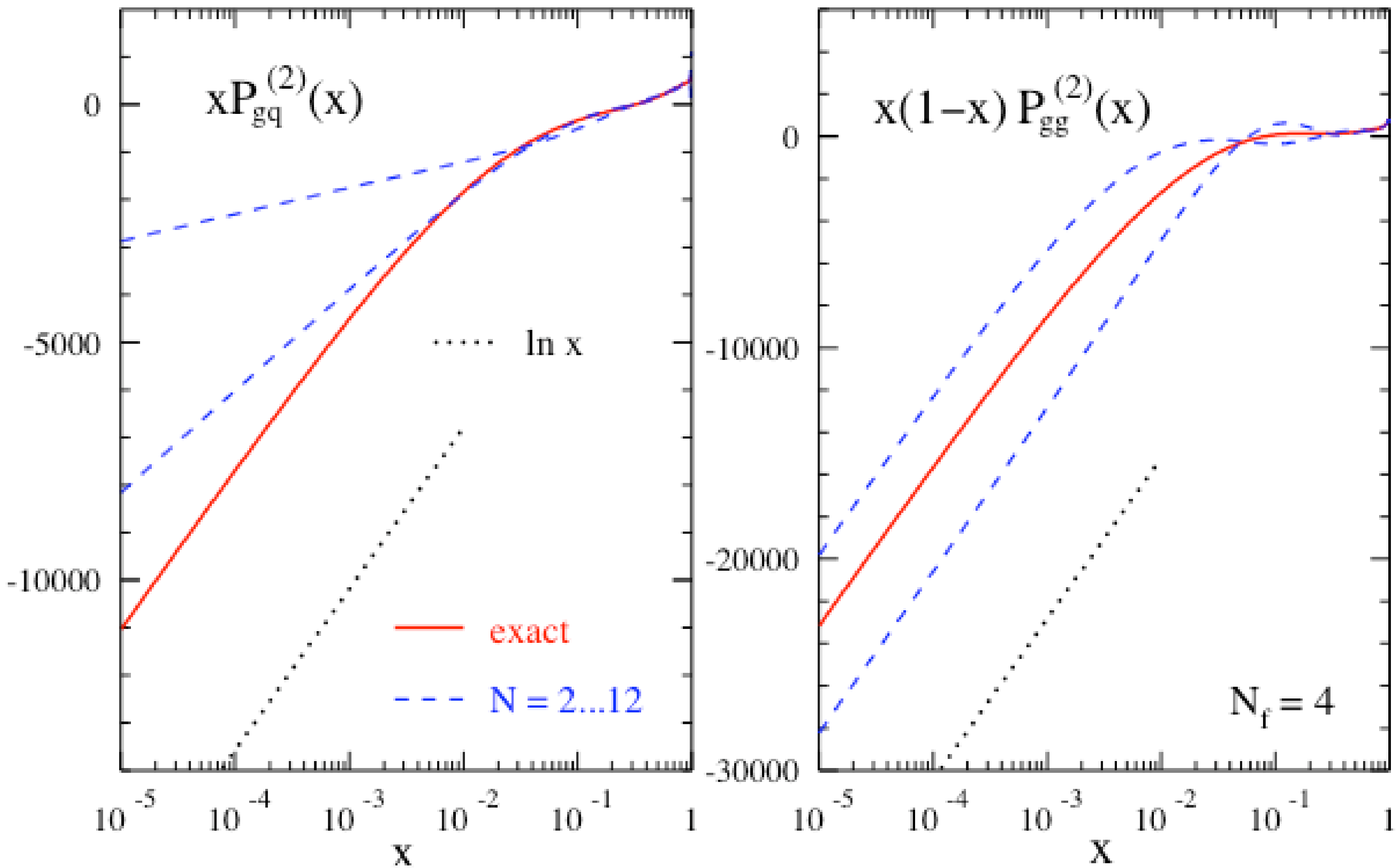
$$P_{qq} = P^{+}_{ns} + n_f (P^{s}_{qq} + P^{s}_{\bar{q}q})$$
$$P_{qg} = n_f P_{q_i g} \,, \quad P_{gq} = P_{g q_i}$$

PDF history

- leading order (or one-loop)
  anomalous dim/splitting functions — Gross Wilczek 1973; Altarelli Parisi 1977
- NLO (or two-loop)
  F_2, F_L — Bardeen Buras Duke Muta 1978
  anomalous dim/splitting functions — Curci Furmanski Petronzio 1980
- NNLO (or three-loop)
  F_2, F_L — Zijlstra van Neerven 1992; Moch Vermaseren 1999
  anomalous dim/splitting functions — Moch Vermaseren Vogt 2004
  - the calculation of the three-loop anomalous dimension is the toughest calculation ever performed in perturbative QCD!
    - one-loop $\gamma_{ij}^{(0)}/P_{ij}^{(0)}$ ⇒ 18 Feynman diagrams
    - two-loop $\gamma_{ij}^{(1)}/P_{ij}^{(1)}$ ⇒ 350 Feynman diagrams
    - three-loop $\gamma_{ij}^{(2)}/P_{ij}^{(2)}$ ⇒ 9607 Feynman diagrams

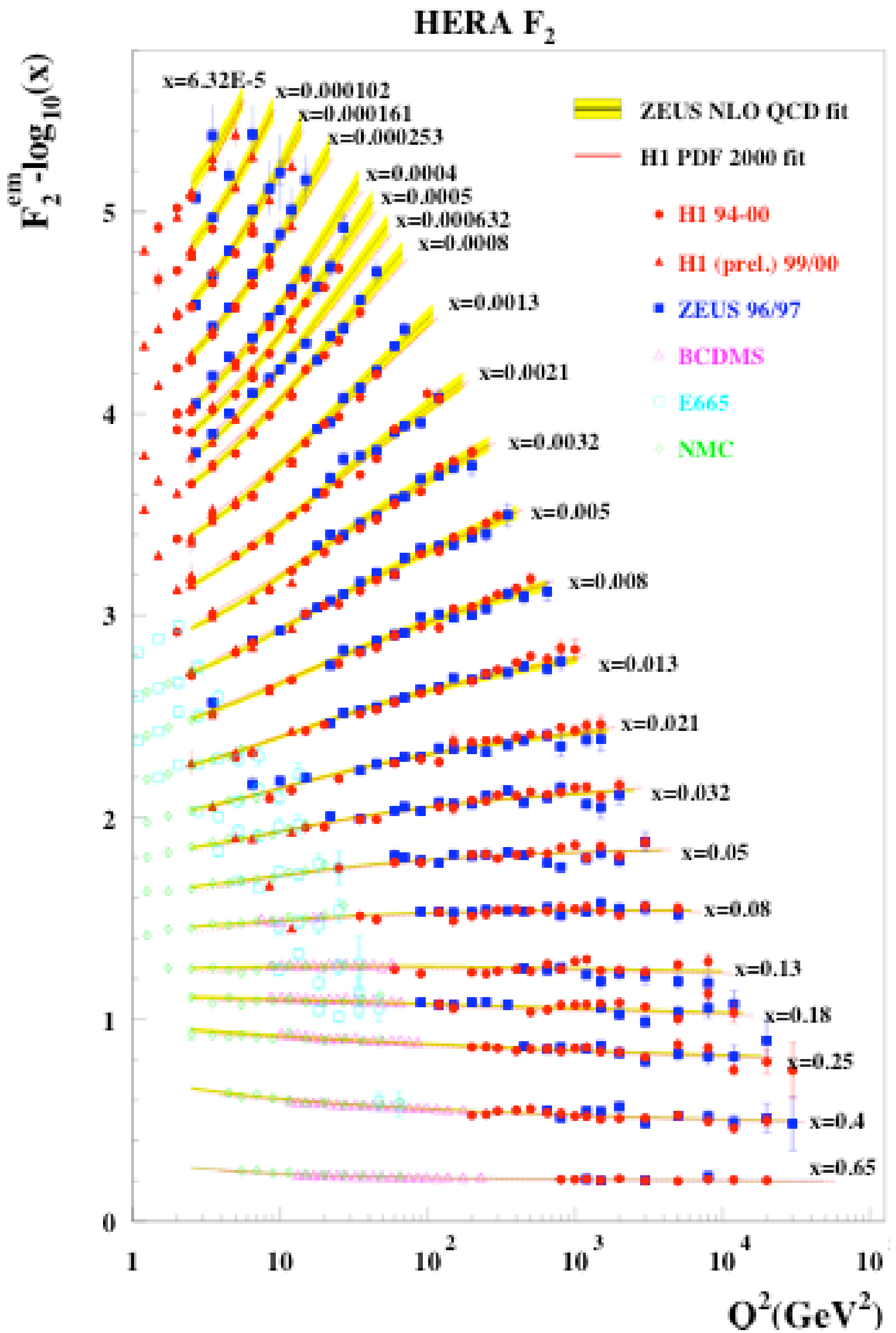
    20 man-year-equivalents, 10^6 lines of dedicated algebra code

Numerical examples



- exact NNLO results, estimates from fixed moments and leading small-x term

HERA F_2



Bjorken-scaling violations

H1, ZEUS: ongoing fits for PDF's; so far NNLO not included

PDF global fits

J. Stirling, KITP collider conf 2004

global fits

MRST: Martin Roberts Stirling Thorne

CTEQ: Pumplin et al.

Alekhin (DIS data only)

method

Perform fit by minimising χ^2 to all data, including both statistical and systematic errors

Start evolution at some Q_0^2, where PDF's are parametrised with functional form, e.g. $xf(x, Q_0^2) = (1-x)^\eta (1 + \epsilon x^{0.5} + \gamma x) x^\delta$

Cut data at $Q^2 > Q^2_{\rm min}$ and at $W^2 > W^2_{\rm min}$ to avoid higher twist contamination

Allow $\bar{u} \neq \bar{d}$ as implied by E866 Drell-Yan asymmetry data

accuracy

NLO evolution
and fixed moments of NNLO

H1, ZEUS $F_2^{e^+p}(x,Q^2)$, $F_2^{e^-p}(x,Q^2)$

BCDMS $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

NMC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2), (F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2))$

SLAC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

E665 $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

CCFR $F_2^{\nu(\bar{\nu})p}(x,Q^2), F_3^{\nu(\bar{\nu})p}(x,Q^2)$

$\rightarrow$ q, $\bar{q}$ at all x and g at medium, small x

H1, ZEUS $F_{2,c}^{e^+p}(x,Q^2) \rightarrow c$

E605, E772, E866 Drell-Yan $pN \rightarrow \mu\bar{\mu} + X \rightarrow \bar{q}\ (g)$

E866 Drell-Yan p,n asymmetry $\rightarrow \bar{u}, \bar{d}$

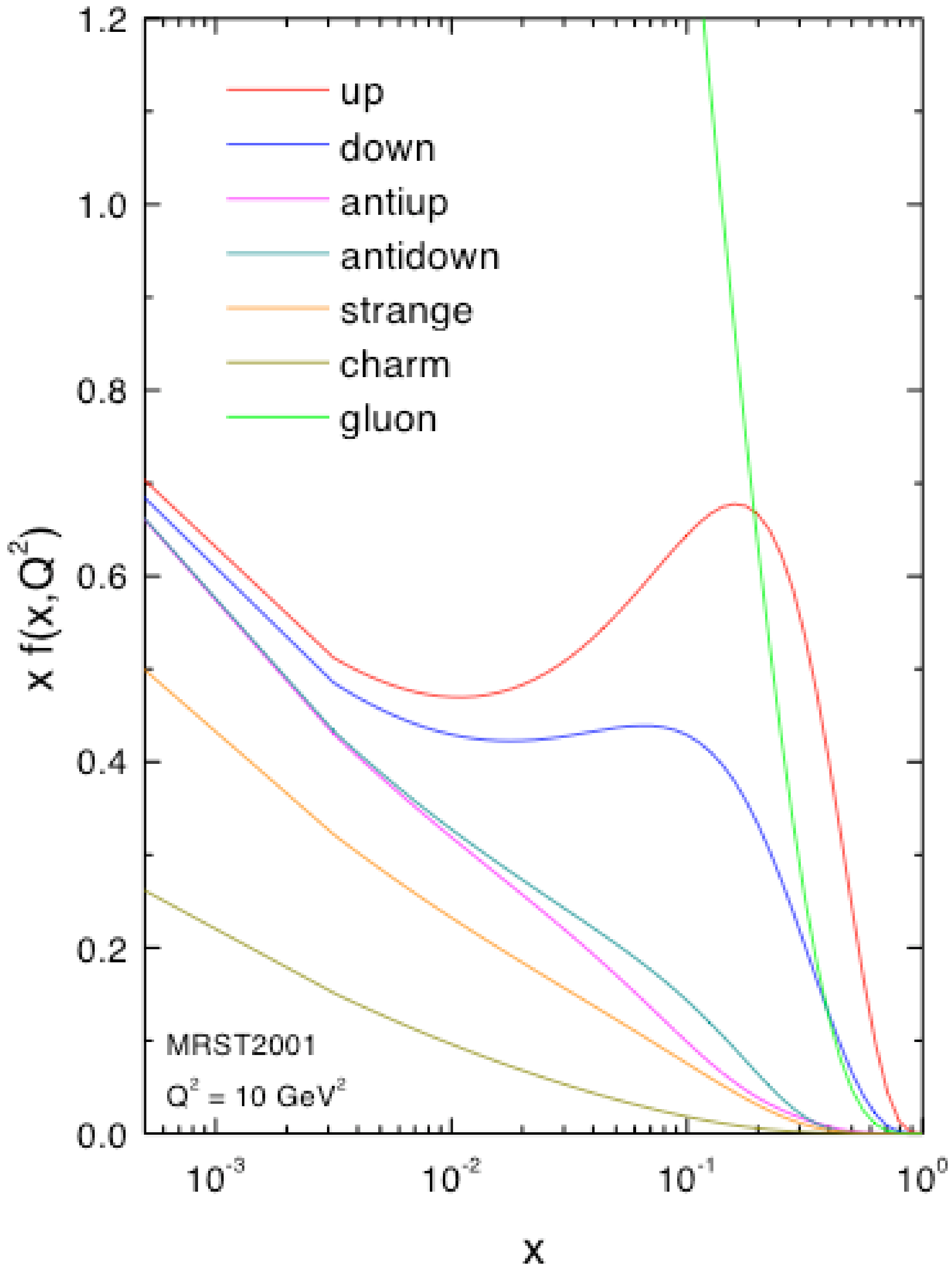
CDF W rapidity asymmetry $\rightarrow u/d$ ratio at high x

CDF, D0 Inclusive jet data $\rightarrow g$ at high x

CCFR, NuTeV Dimuon data constrains strange sea s, $\bar{s}$

- no prompt photon data included in the fits

MRST 2001 PDF's



PDF uncertainties

- direct effect on Tevatron & LHC cross section predictions
- various approaches being used, most notably
  - Hessian (error matrix) approach (H1, ZEUS, CTEQ, Alekhin)

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j}(a_i - a_i^{(0)})H_{ij}(a_j - a_j^{(0)})$$

H is related to the covariance matrix of the parameters $C_{ij}(a) = \Delta\chi^2 (H^{-1})_{ij}$

diagonalise H_{ij} and define PDF sets $S_i^{\pm}$ displaced along the eigenvector direction by $\Delta\chi^2 = \sum_i z_i^2$. Then uncertainty on physical quantity is given by

$$(\Delta F)^2 = \frac{1}{2}\sum_i \left(F(S_i^{(+)}) - F(S_i^{(-)})\right)^2$$

  - Lagrange multiplier method (CTEQ, MRST)

perform fit while constraining value of some physical quantity F. Minimise

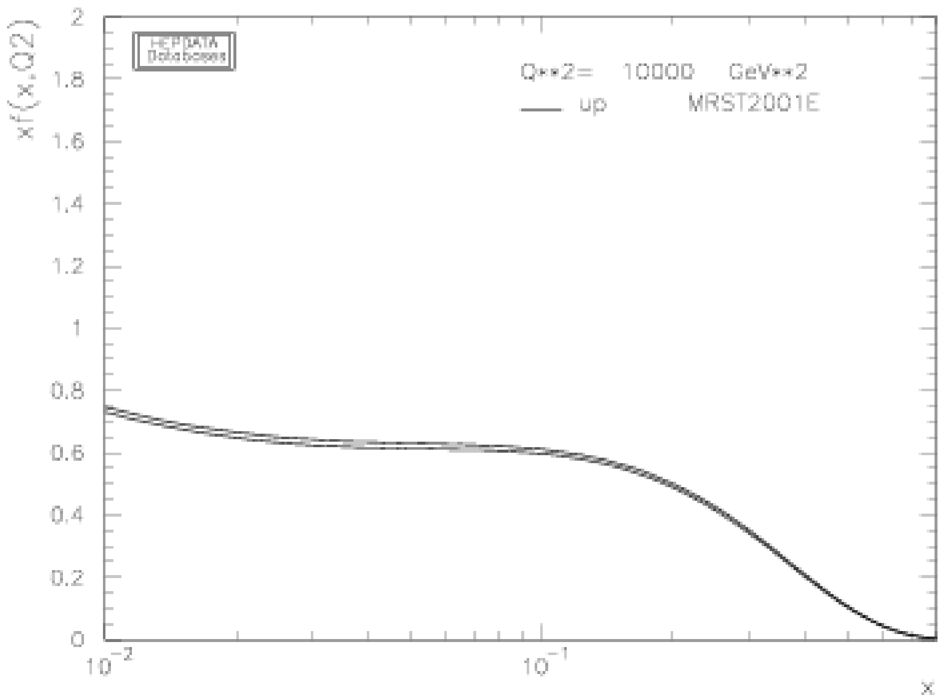
$$\Psi(\lambda, a) = \chi^2_{\rm global}(a) + \lambda F(a)$$

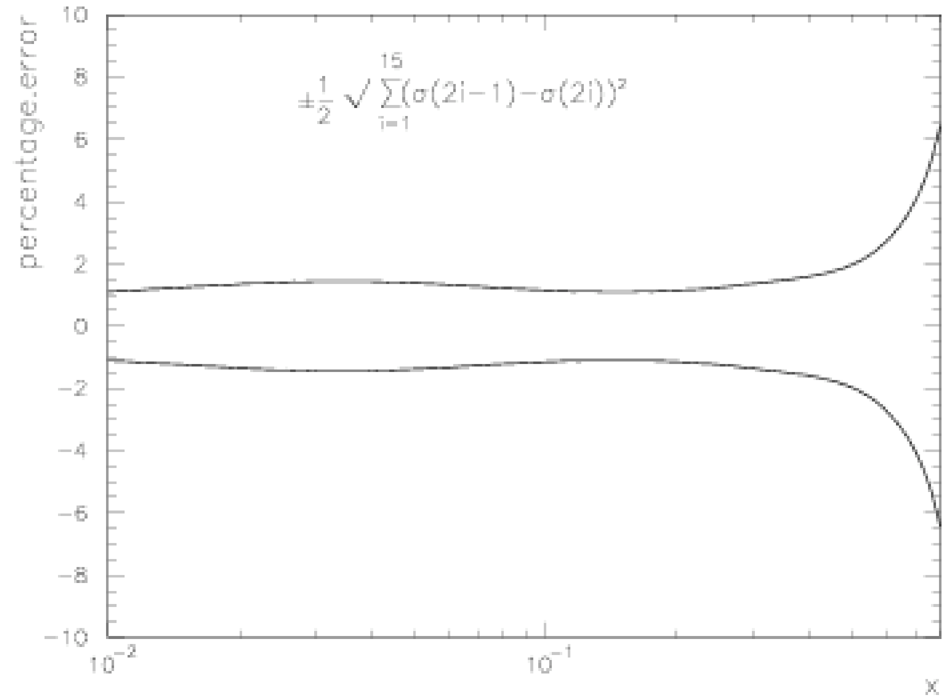
for various values of λ and parton parameters $\{a\}$. Gives set of best fits for particular values of parameter $F(a)$. Uncertainty then determined by deciding allowed range of $\Delta\chi^2$. Can also see which data sets in global fit most directly influenced by variation in $F(a)$

want to know more ? see e.g. R. Thorne et al hep-ex/0205233

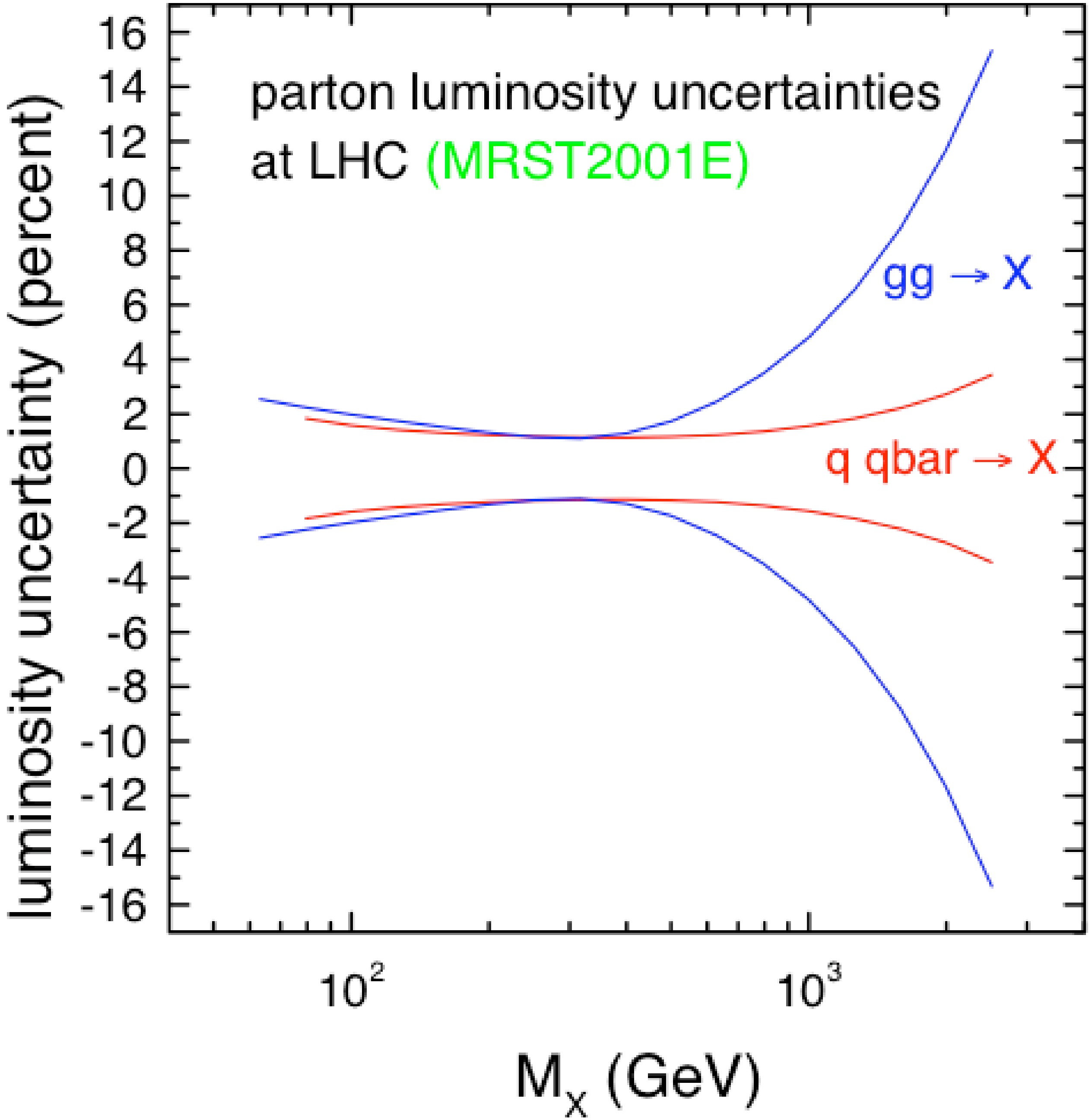
Error on up distribution at $Q^2 = 10000 \text{ GeV}^2$

from MRST2001E (see hep-ph/0211080)



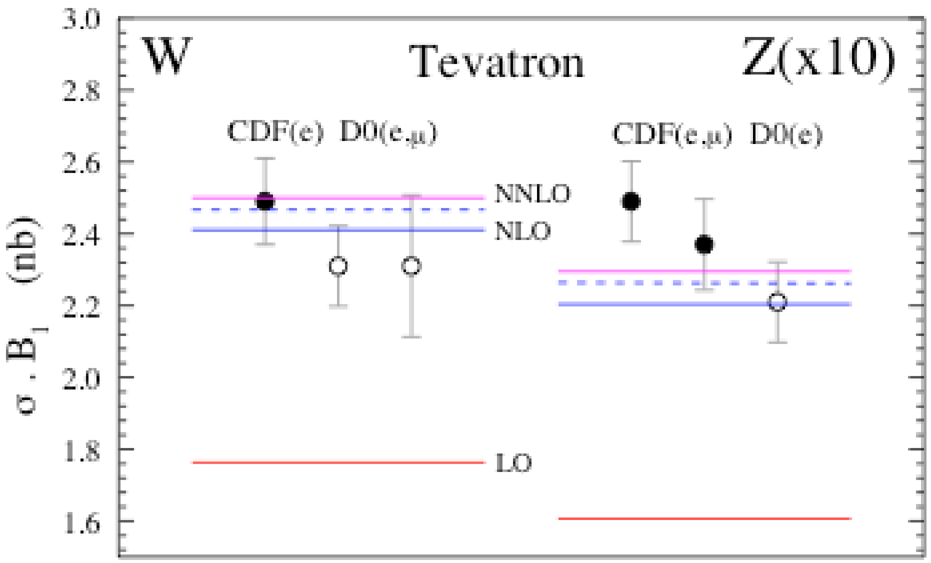


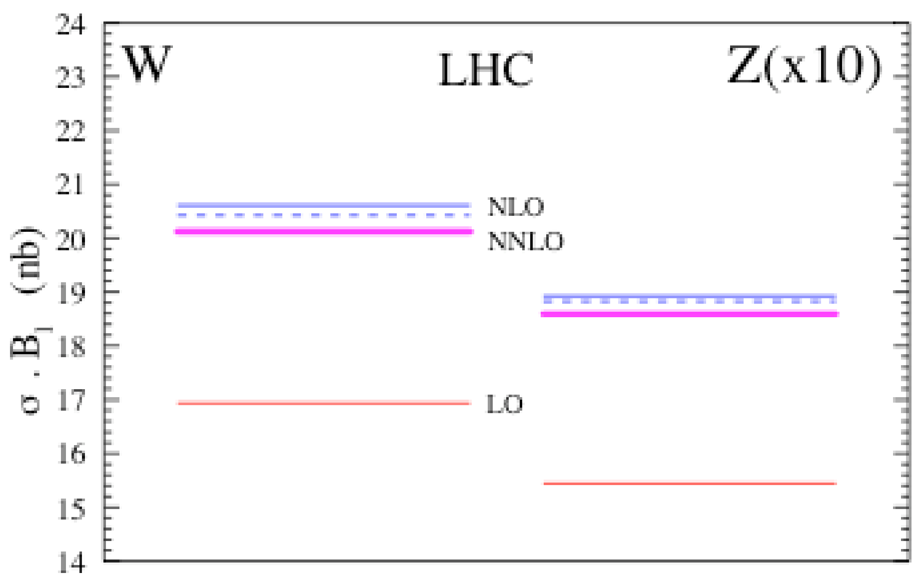
- Hessian method used
- error is about 2%



- for $q\bar{q}$ (relevant for Drell-Yan production)
- for gg (relevant for Higgs production)

W, Z total cross sections



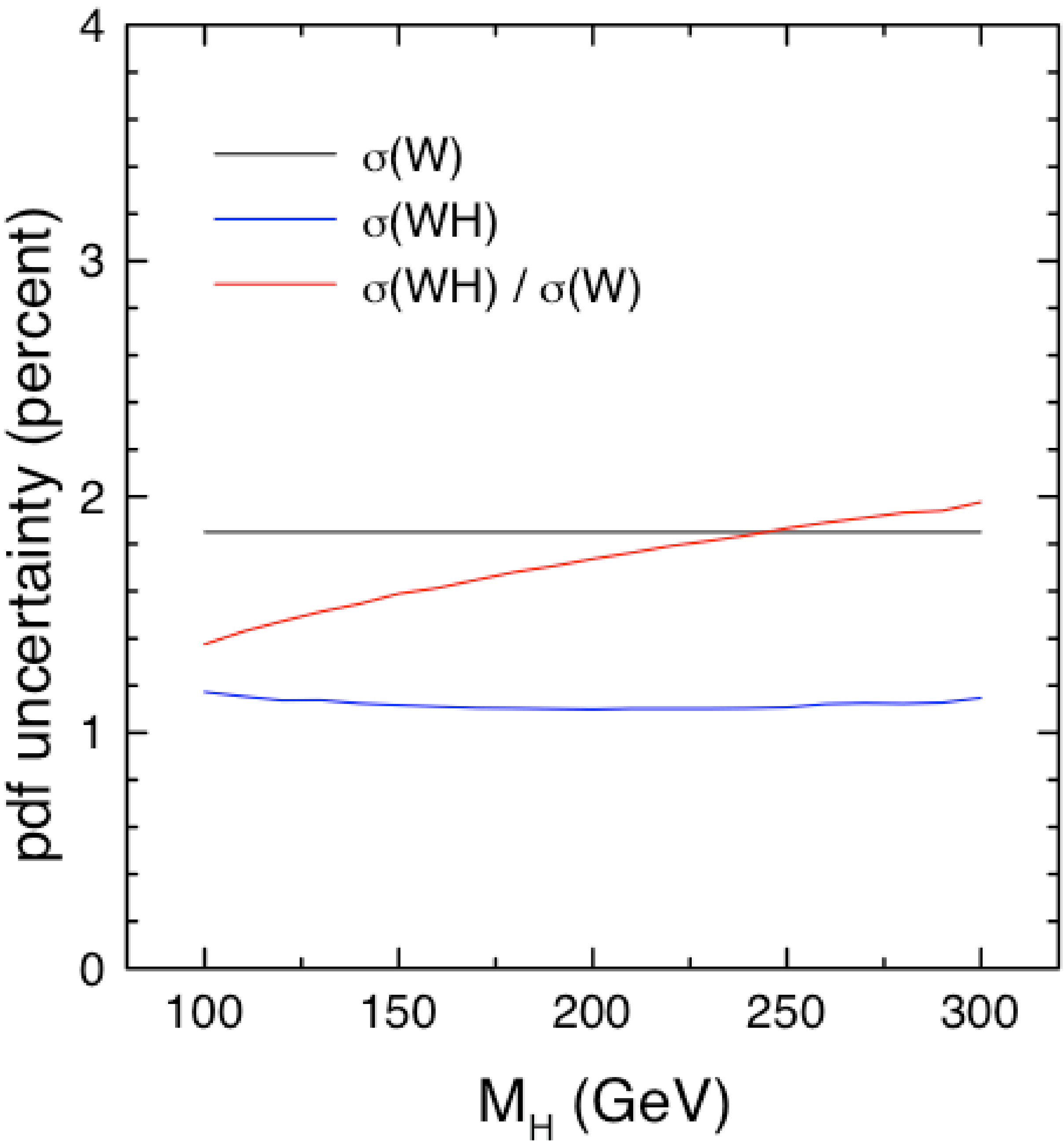


- MRST2001
- NNLO: only few fixed moments
- current best (MRST) estimate

  $$\delta\sigma_{W,Z}^{\mathrm{NNLO}}(\mathrm{total\ pdf}) = \pm 4\%$$

  (expt. pdf error is 2%)
- larger uncertainty in the NLO prediction, because of problems at small x in the global fit to DIS data and because large rapidity W, Z's sample small x

PDF uncertainty on W, WH cross sections at LHC

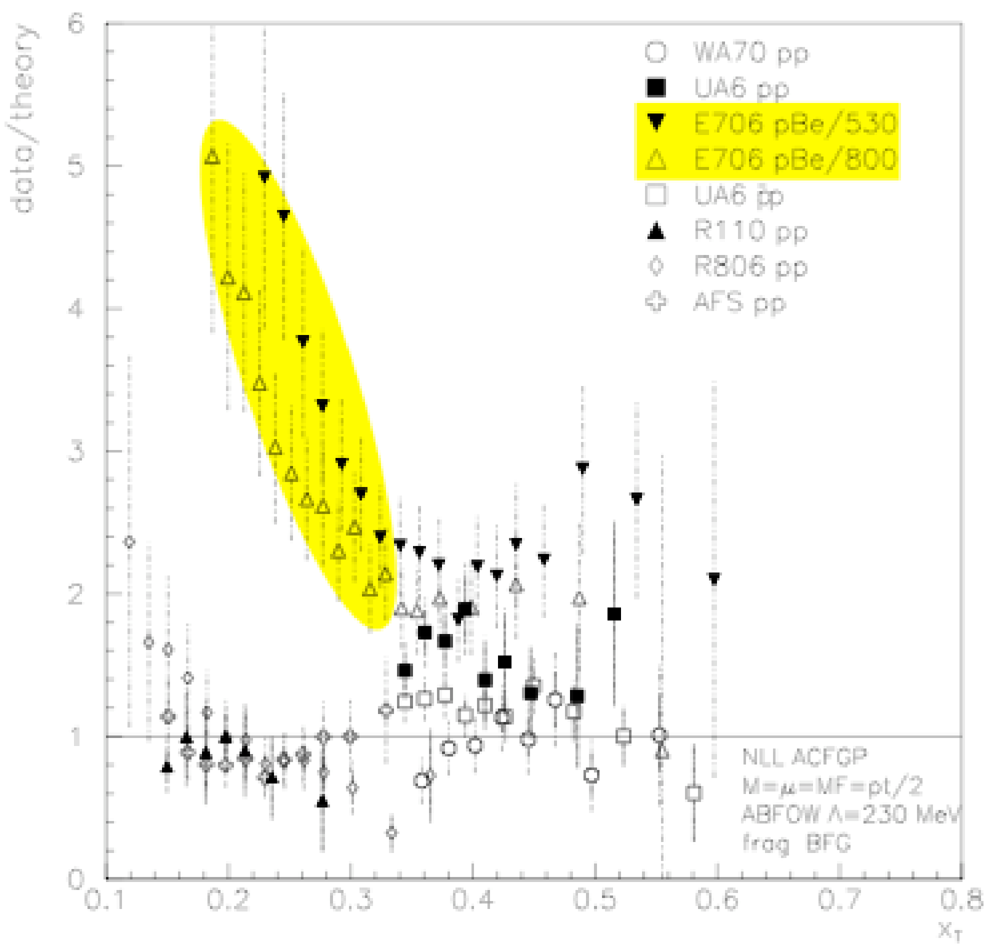


- MRST2001E
- use $\sigma(W), \sigma(Z)$ as ``standard candles'', i.e. to calibrate other cross sections, e.g. $\sigma(WH)$
- $\sigma(WH)$ more precisely predicted because it samples quark PDF's at higher x than $\sigma(W)$

Hinc sunt photones

Photons at fixed-target experiments

- probe the gluon distribution at high x
  at $\sqrt{s} = 1800\ \mathrm{GeV}, \quad p_{Tjet} = 180\ \mathrm{GeV}$ ⇒ $x_T = 0.2$

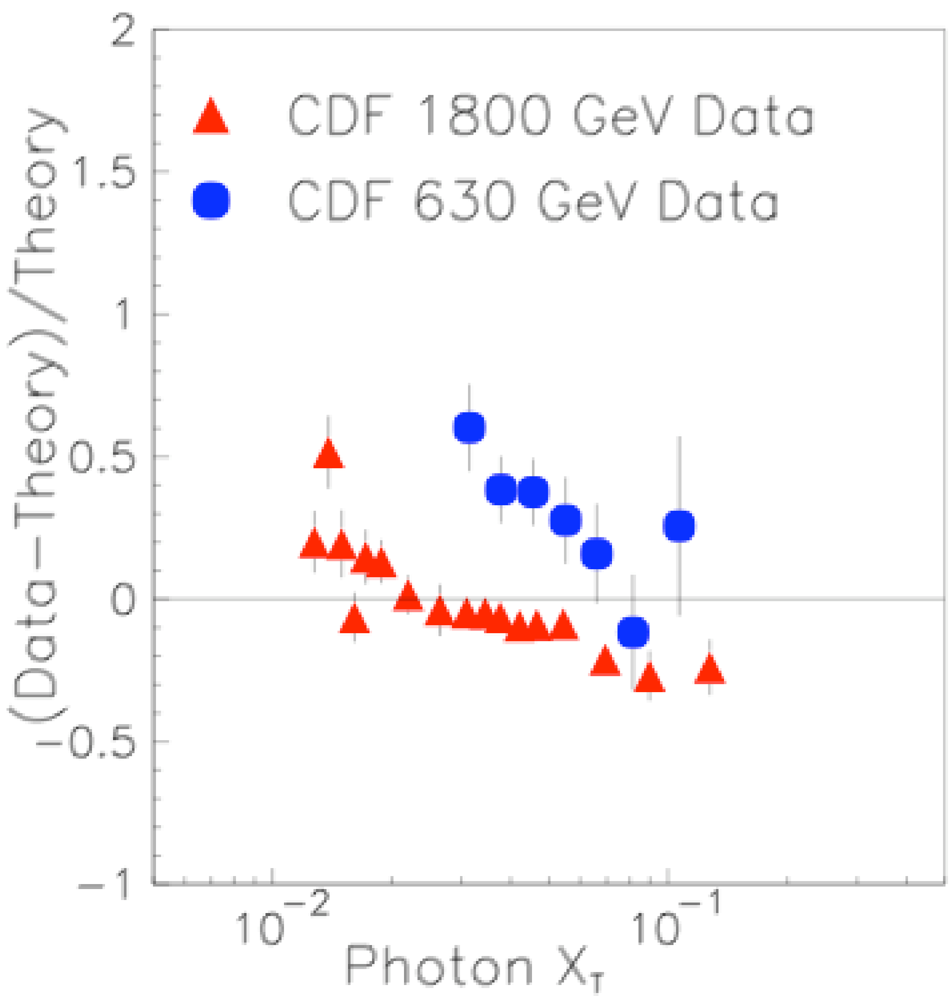


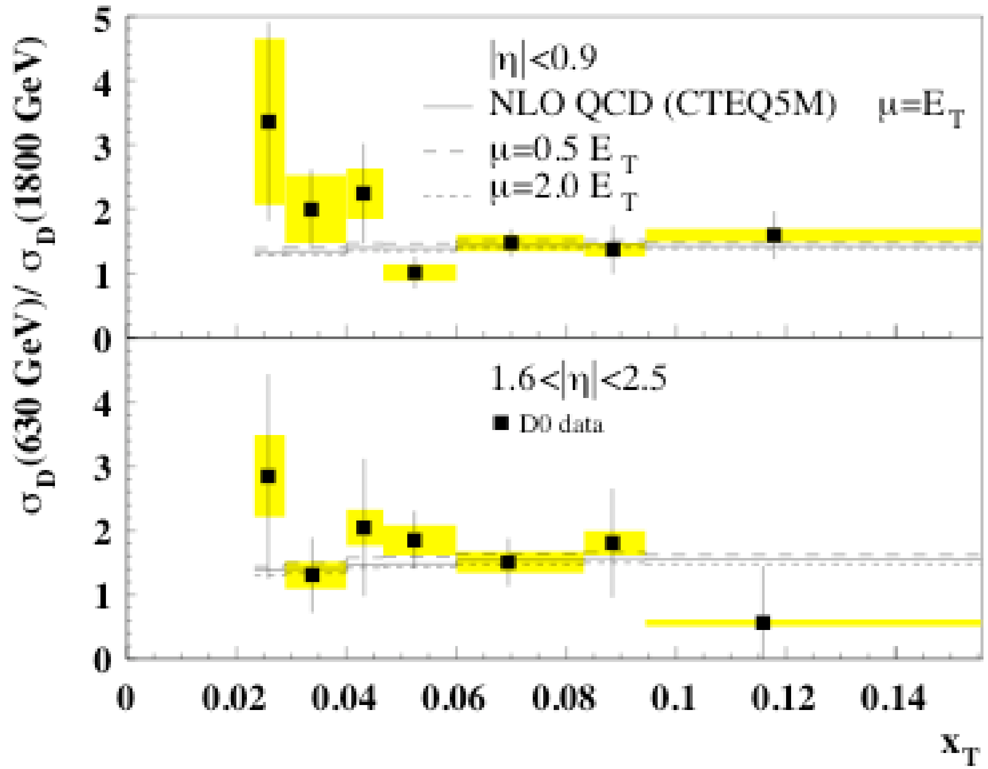
P. Aurenche et al. 1998

- data are not consistent with theory, and (even more worrisome) are not consistent with each other

- currently they are not used in PDF fits

Photons at the Tevatron at 1800 GeV and 630 GeV

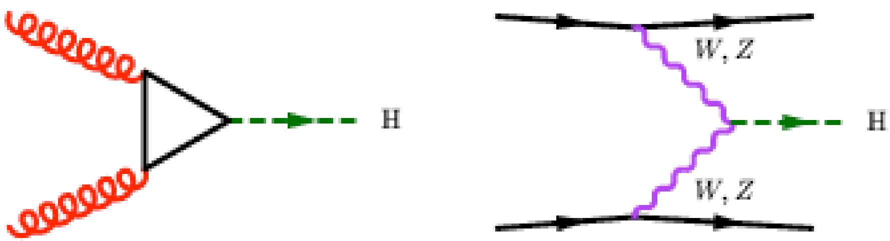




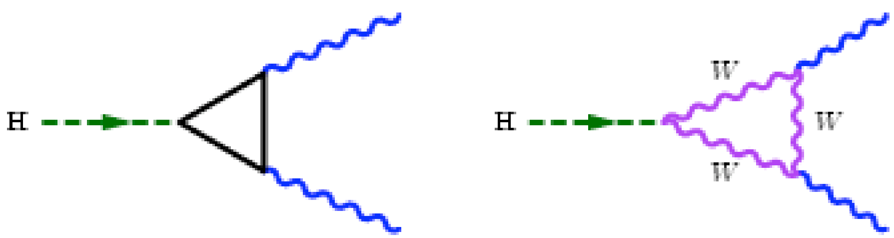
- data are not consistent with theory (but D0 is better off than CDF)
- Problems ? TH: Narrow isolation cones used by experiments

Photons as a background to Higgs searches

- Higgs production



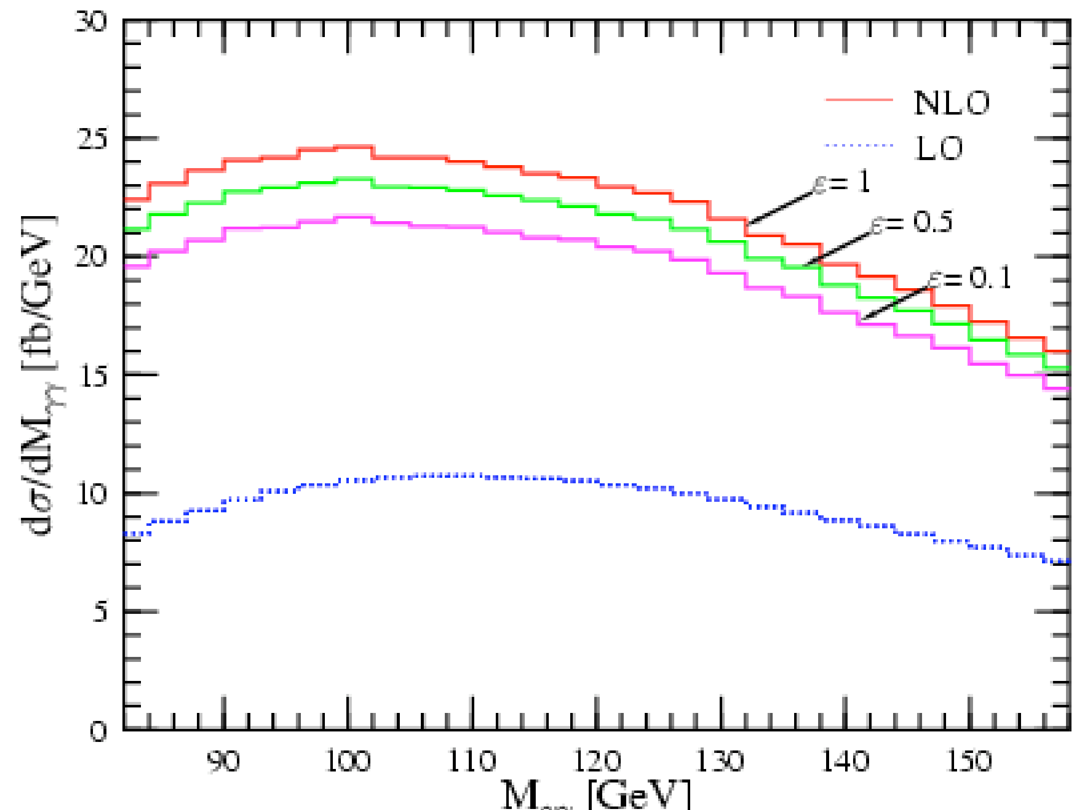
- Higgs decay



- Di-photon decay important in the low-mass Higgs searches

$pp \to \gamma\gamma + \text{jet}$ at LHC

di-photon invariant mass distribution



F. Maltoni Z. Nagy Z. Trocsanyi VDD 2003

- isolation cone $R_\gamma = 0.4$
  hadronic energy allowed inside cone is $E_{T,\max} = \epsilon\, p_{T\gamma}$
- used Frixione's photon isolation criterion (avoids use of fragmentation functions)

$$E_T \le E_{T,max} \left(\frac{1-\cos r}{1-\cos R_\gamma} \right)^n$$

- K factor is > 2

Conclusions

- QCD is an extensively developed and tested gauge theory
- a lot of progress in the last 4-5 years in
  - MonteCarlo generators
  - NLO cross sections with one more jet
  - NNLO computations
- better and better approximations of signal and background for Higgs and New Physics
- new formal developments (I didn't discuss): QCD as a string theory in twistor space E. Witten 2003
  - novel ways of computing (analytically) tree multi-parton matrix elements and (N=4) loop matrix elements F. Cachazo P. Svrcek E. Witten 2004